Production Externalities, Environmental Taxes, and the Gains from Trade

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Production Externalities, Environmental Taxes, and the Gains from Trade

Soham Baksi\textsuperscript{1} and Michael Benarroch\textsuperscript{2}

Abstract
We analyze the effects of environmental taxation on the pattern of and gains from trade in a two-country Ricardian framework, where production in a polluting sector (e.g. manufacturing) adversely affects productivity in an environmentally sensitive sector (e.g. agriculture). The two countries differ in terms of their production technology so that the productivity loss suffered by the environmentally sensitive sector is higher in the dirtier country. When the countries do not pursue any environmental policy, the dirtier country has a comparative advantage in the polluting good and exports that good in the trading equilibrium. If preference for the polluting good is low, the dirtier country loses from trade while its trading partner gains. Global gains from trade are also negative as the market determined pattern of trade is inefficient. Introduction of a unilateral pollution tax by the dirtier country can enable it to reverse the pattern of trade and the distribution of the gains from trade, such that international trade becomes welfare-improving for that country as well as globally. The conventional pollution haven result may get reversed in the presence of cross-sectoral externalities, as each country has an incentive to set the tax such that it exports the good that is more preferred by consumers.

\textit{JEL classifications:} Q56; F18

\textit{Keywords:} Ricardian model; Production externality; Pollution tax

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1 Introduction

There is a growing literature that examines the linkages between international trade, environmental policy and pollution. The theoretical literature has shown that differential environmental policies across countries can lead to a “pollution haven effect” where countries with weaker environmental regulations have a competitive advantage in the production of dirty goods (e.g. Copeland and Taylor, 2004; Levinson and Taylor, 2008). Throughout most of this literature, it is assumed that the damaging effects of pollution occur as a disutility cost for consumers.³

There are many situations, however, where pollution generated by one sector negatively affects production in another sector of the economy. For example, the degradation of air, land or water bodies due to manufacturing and mining activities are often observed to adversely affect agriculture, forestry, fisheries and the tourism sectors.⁴ A smaller body of literature considers this second effect of pollution and analyzes its implications for international trade (Copeland and Taylor, 1999; Benarroach and Thille, 2001; Unterroberdoerster, 2001; Zeng and Zhao, 2009; Rus, 2010). This literature finds that, when pollution negatively affects other producers rather than consumers, international trade can lead to a spatial separation of production across countries, as in the pollution haven literature, even in the absence of differential environmental policies across countries. Moreover, the market determined pattern of trade can be inefficient and trade can lead to welfare losses. The role of environmental regulations in correcting such market failures involving cross-sectoral externalities, however, remains to be analyzed in the literature.

In this paper, we develop a two-country Ricardian model of international trade and pollution, where local pollution generated through production in a dirty industry harms productivity in an environmentally sensitive industry. The two countries differ in terms of their production technology, so that the productivity loss suffered by the environmentally sensitive industry is higher in the dirtier country. Our model extends Copeland and Taylor

³In practice, pollution generated through production or consumption activities can affect the utility functions of consumers (i.e. “consumption externality”) and/or the production functions of producers (i.e. “production externality”).

⁴Evidence suggests that these external costs are substantial for many countries (e.g. see Pearce and Warford, 1993). According to Environment Canada, the impacts of ground level ozone on agriculture are known to cost Canadian farmers millions of dollars in lost production each year. They also note that, “At current acid rain levels, over half a million cubic metres of wood are being lost from forests in Atlantic Canada due to reduced soil nutrients and tree growth. At market prices, the value of this lost wood is in the hundreds of millions of dollars. Based on soil content and acid rain levels in central Canada, it is likely that millions of cubic metres of wood are also being lost in Ontario and Quebec each year... Acid rain can also significantly reduce fish stocks in Canada which could have a serious implication for both recreational fishing and Canada’s multi-billion-dollar per year commercial fishing industry.” (see www.ec.gc.ca/air/default.asp?lang=En&n=7DBE4D9F-1)
(1999), Benaroch and Thille (2001) and others by examining the effects of an environmental tax imposed by the dirtier country on the pattern of and gains from trade. By constructing the model in this form, we limit the applicability of our study to cases where one country has a relatively stringent environmental regulation while its trading partner is much more permissive and passive. This, however, allows us to focus directly on the effects of differential environmental policies on the pattern of comparative advantage and welfare. To our knowledge, ours is the first paper that considers environmental regulation in a model of international trade where pollution generated by one sector lowers productivity in another sector of the economy.

We first employ a benchmark model where, in the absence of any regulation, the dirtier country with the more severe production externality problem has a comparative advantage in the polluting good and exports that good in the trading equilibrium. If preference for the polluting good is low, this country diversifies and loses from trade. Because this pattern of production is globally inefficient, the global gains from trade can also be negative. We then extend the literature by introducing the possibility that the dirtier country imposes a tax on its polluting sector. We show that imposition of the pollution tax under artarky enables the dirtier country to raise the relative price of the polluting good and reverse its comparative advantage such that, when trade opens up, it exports the clean good. This reversal of the pattern of trade not only enables the dirtier country to gain from trade when preference for the polluting good is low, but also converts the global pattern of production into an efficient one. As a result, the global production possibility frontier shifts up and the global gains from trade increase.

Irrespective of whether the dirtier country is large or small, we find that the country can be better off in the trading equilibrium by unilaterally imposing a pollution tax, at a sufficiently high rate so as to drive out the polluting industry from its borders, and specializing in the clean good. This happens if the world relative price of the polluting good is low (for a small country) or if preference for the polluting good is low (for a large country). This incentive for unilateral taxation in the presence of cross-sectoral externalities consti-

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5From a policy perspective, the issue of unilateral environmental taxation is relevant with respect to several countries (e.g. EU member states) and has been analyzed by other authors (e.g. Markusen, et al., 1993; Barker, et al., 2007).

6An exception is Zeng and Zhao (2009), who examine the pollution haven hypothesis using a monopolistic competition model of trade with production externalities, increasing returns and international mobility of capital. They find that pollution havens may not arise in smaller countries with weaker environmental regulations if the cost-reducing effect of weaker regulations is dominated by the demand-reducing effect of pollution or the forces of agglomeration in larger countries. However, the environmental regulations are assumed to be exogenously given in their model.

7Copeland and Taylor (1999) and Benaroch and Thille (2001) derive similar results.
tutes an interesting reversal of the usual pollution haven result reported in the literature, where countries have an incentive to reduce their environmental taxes in an effort to gain a competitive advantage over their trading partners. Rather than a race to the bottom, the presence of cross-sectoral externalities may motivate a race to the top.

Our findings are similar in spirit to Ethier and Ruffin (2009), who obtain a reversal of comparative advantage through increasing returns to scale external to the firms. As noted in the literature, both cross-sectoral production externalities and external economies of scale can lead to a non-convex production possibility set and serve as a source of gains from trade through the spatial separation (respectively, agglomeration) of mutually incompatible (respectively, reinforcing) industries (Baumol and Bradford, 1972; Panagaria, 1981; Ethier, 1982). Using a Ricardian model, Ethier and Ruffin (2009) show that “advantage reversal” can occur when scale economies dominate differences in comparative costs across countries. In such situations, tariffs can be used to effect a regime change and reverse the pattern of trade. The authors cite this as a reason for the import-substituting industrial policy pursued by several developing countries in the mid-twentieth century. As we show below, in the presence of production externalities, unilateral environmental taxation by the dirtier countries can be used to bring about a similar beneficial reversal in the pattern of trade.

The remainder of the paper is organized as follows. Section 2 introduces the model, and the autarky equilibrium is derived in Section 3. Section 4 presents the trading equilibrium for a small open economy, where the pollution tax does not have any terms of trade effect. The equilibrium when two large countries trade with each other is analyzed in section 5, while the next section provides a numerical example. We conclude the paper in section 7.

2 The model

We develop a static Ricardian model of trade similar to that employed in Benaroch and Thille (2001). The model allows for two countries, \( i = A, B \) and two goods, \( j = 1, 2 \) produced using one composite input, labour \( L \). We assume that each country has the same amount of labour \( L = 1 \). In each country, there are two perfectly competitive industries producing the two goods with linear technologies. The production function describing the technology in sector 1 is given by

\[
q_1^i = L_1^i, \tag{1}
\]

where \( q_1^i \) and \( L_1^i \) denote output produced and labour used in the production of good 1 by country \( i = A, B \). With appropriate definition of units, one unit of labour produces one unit of output in industry 1.
Similar to Copeland and Taylor (1999), we consider the effects of local pollution only. Production of good 1 in country A is assumed to generate pollution which negatively affects the productivity of labour in sector 2 within that country. Production of good 2 in country A is given by

\[ q_2^A = (1 - \beta q_1^A) L_2^A, \]  

where \( \beta \in (0, 1) \) is the pollution damage parameter and \( L_1^A + L_2^A = 1 \) is total labour used in that country. The parameter \( \beta \) denotes the extent of the negative externality imposed by the production of the polluting good 1 on the production of the environmentally-sensitive good 2. While the marginal product of labour in sector 2 is decreasing in sector 1’s output, we assume that labour productivity in sector 2 cannot be driven down to zero, i.e. \((1 - \beta L) > 0\).

Country B differs from country A in terms of the extent to which domestic production of good 1 affects labour productivity in sector 2. For expositional simplicity, we assume that the pollution damage parameter takes the value of zero in country B.\(^8\) Thus, production of good 2 in country B is given by

\[ q_2^B = L_2^B, \]  

where \( L_1^B + L_2^B = 1 \).

With these production conditions, we have \( q_1^j \in [0, 1] \). Given the linear production technologies in country B, its production possibility frontier (PPF) is also linear, \( q_2^B = 1 - q_1^B \). On the other hand, the externality imposed by sector 1 on the productivity of labour in sector 2 results in a convex PPF in country A, \( q_2^A = (1 - q_1^A) (1 - \beta q_1^A) \). Figure 1 shows the PPF in each country. The slope of the production frontier in A is characterized by

\[ \frac{dq_2^A}{dq_1^A} = -(1 - \beta q_1^A) - \beta (1 - q_1^A) < 0, \quad \frac{d^2 q_2^A}{d(q_1^A)^2} = 2 \beta > 0 \]

The convex PPF leads to a non-convex production possibilities set in country A.

Consumers are assumed to have the same preferences regardless of location. Utility derived by the representative consumer in each country is given by

\[ u \left( x_1^i, x_2^i \right) = b \ln x_1^i + (1 - b) \ln x_2^i, \]  

where \( x_j^i \) is the amount of good \( j = 1, 2 \) consumed by the representative consumer in

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\(^8\)This assumption is relaxed in section 6. The damaging effects of pollution can differ across countries due to technological or geographical reasons. For example, production of good 1 in country B may involve a cleaner technology, or the pollution generated by good 1 may be transported out of country B by winds or rivers flowing in a favourable direction.
country \( i = A, B \), and \( b \in (0, 1) \) is a parameter denoting the relative preference for good 1. An implication of this utility function is that consumers must consume positive amounts of both goods in equilibrium, as \( \lim_{x_j^* \to 0} \frac{q_{x_j^*}}{x_j^*} \to \infty \).

3 Autarky equilibrium

In this section, we examine the autarky equilibrium where each country has a closed economy. The next section allows for international trade. Throughout the paper, we consider a world where country A may impose a pollution tax on domestic producers of good 1, while country B does not pursue any such policy.

3.1 Equilibrium in country B

First consider country B, which does not tax the production of good 1. By maximizing (4) subject to B’s budget constraint, \( w^B = p_1^B x_1^B + p_2^B x_2^B \), we obtain the demand for goods 1 and 2 in country B as

\[
x_1^B = \frac{bw^B}{p_1^B}, \quad x_2^B = \frac{(1 - b) w^B}{p_2^B}
\]

Equation (5) shows that demand for each good is positively related to the share of total expenditure spent on each good, \( b \) and \( (1 - b) \), national income, \( w^B \), and negatively related to the prices, \( p_1^B \) and \( p_2^B \).

To derive the equilibrium under autarky, we assume full employment, use (1) and (3) to determine supply, (5) to determine demand, and set domestic demand equal to supply. Given perfect competition, the wage in country B is equal to the value of the marginal product of labour. This implies real wages are

\[
\frac{w^B}{p_1^B} = 1, \quad \frac{w^B}{p_2^B} = 1
\]

Substituting (6) in (5), we get the equilibrium consumption and production of each good

\[
\tilde{x}_1^B = \tilde{q}_1^B = b, \quad \tilde{x}_2^B = \tilde{q}_2^B = 1 - b
\]

where a tilde (\( \tilde{} \)) over a variable denotes its value in the autarky equilibrium. The price of good 1 relative to that of good 2 in country B under autarky, obtained by taking the ratio of real wages in (6), is

\[
\tilde{p}^B = \frac{\tilde{p}_1^B}{\tilde{p}_2^B} = 1
\]
Substituting (7) into (4) yields the autarky equilibrium utility level derived by the representative consumer in country B as

\[ \tilde{u}^B = b \ln b + (1 - b) \ln (1 - b) \]  

(8)

### 3.2 Equilibrium in country A

Next consider country A, which accounts for the externality by imposing a tax \( \tau \geq 0 \) per unit of the polluting good produced domestically. We assume that trade agreements or other legal or political constraints prevent country A from subsidizing producers of the polluting good so that \( \tau \) is non-negative. The profit maximization problem for competitive firms in industry 1 is then given by

\[
\max_{L_1^A} \Pi_1^A = p_1^A q_1^A - w^A L_1^A - \tau q_1^A
\]

Substituting for \( q_1^A \) using the production function (1) and solving yields the following zero profit condition

\[
\frac{w^A}{p_1^A} = 1 - \frac{\tau}{p_1^A} = 1 - t,
\]

(9)

where \( t \equiv \frac{\tau}{p_1^A} \) is the advalorem equivalent of the unit tax \( \tau \). Since the real wage in country A, \( \frac{w^A}{p_1^A} \), must be positive, we have \( t \in [0, 1) \). Further, the zero profit condition in the production of good 2 implies

\[
\frac{w^A}{p_2^A} = 1 - \beta q_1^A
\]

(10)

Introduction of the environmental tax raises the cost of producing good 1 and, as shown below, lowers the quantity of good 1 produced in the autarky equilibrium. The utility maximization problem now must account for the tax revenue that is collected and remitted back to consumers. Consumers in country A also maximize utility given by (4), but face the following budget constraint

\[
w^A + \tau q_1^A = p_1^A x_1 + p_2^A x_2,
\]

where \( w^A + \tau q_1^A \) is labour income plus the tax revenue remitted back to consumers. Con-
sequently, demand for goods 1 and 2 in country A are

\[ x_1^A = \frac{b(w^A + \tau q_1^A)}{p_1^A} = \frac{b(w^A + t q_1^A)}{p_1^A} \]
\[ x_2^A = \frac{(1 - b)(w^A + \tau q_1^A)}{p_2^A} = p^A(1 - b)\left(\frac{w^A}{p_1^A} + t q_1^A\right) \]

where \( p^A \equiv \frac{p_1^A}{p_2^A} \). Using (9)-(11), and setting domestic supply equal to demand, the equilibrium output of goods 1 and 2 under autarky are derived as

\[ \tilde{x}_1^A (t) = \tilde{q}_1^A (t) = \frac{b(1 - t)}{1 - bt}, \quad \tilde{x}_2^A (t) = \tilde{q}_2^A (t) = \frac{(1 - b)(1 - b\beta - bt (1 - \beta))}{(1 - bt)^2} \]

Moreover, using (9), (10) and (12), we obtain the autarkic relative price ratio in country A

\[ \tilde{p}^A (t) = \frac{\tilde{p}_1^A}{\tilde{p}_2^A} = \frac{1 - \beta \tilde{q}_1^A}{1 - t} = \frac{1 - b\beta - bt (1 - \beta)}{(1 - t)(1 - bt)} \]

Given that \( b \in (0, 1) \), \( \beta \in (0, 1) \) and \( t \in [0, 1) \), the quantities in (12) are positive and output of good 1 (respectively, good 2) is negatively (respectively, positively) related to the environmental tax. As a result, we find that \( \frac{d\tilde{p}^A(t)}{dt} > 0 \) and Lemma 1 holds.

**Lemma 1:** In the autarkic equilibrium, an increase in the tax imposed on the polluting good by country A causes its relative price to rise.

Two factors contribute to this result. A higher environmental tax (i) raises the marginal cost of producing good 1, and (ii) lowers output of good 1, which raises labour productivity in sector 2. In particular, we have \( \tilde{p}^A (t) \geq \tilde{p}^B = 1 \) if and only if \( t \geq t^* \), where \( 0 < t^* \equiv \frac{1}{2b} \left(1 + b\beta - \sqrt{(1 + b\beta)^2 - 4b^2\beta}\right) < 1 \).

Substituting (12) in (4) yields the utility level for the representative consumer in country A

\[ \tilde{u}^A (t) = \tilde{u}^B + b \ln \frac{1 - t}{1 - bt} + (1 - b) \ln \frac{1 - b\beta - bt (1 - \beta)}{(1 - bt)^2} \]

Note that \( \tilde{u}^A (t) < \tilde{u}^B \) for all \( b \in (0, 1) \), \( \beta \in (0, 1) \) and \( t \in [0, 1) \), where \( \tilde{u}^B \) is given by (8). Due to the detrimental effect of pollution, consumers in country A are worse off than those in country B.
3.2.1 Optimal tax in a closed economy

Solving the first order condition, \( \frac{d\tilde{x}_{A}(t)}{dt} = 0 \), country A’s optimal environmental tax, which maximizes its welfare under autarky, is obtained as\(^9\)

\[
\tilde{t}(b, \beta) = \frac{1 + \beta - 2b\beta - \sqrt{(1 + \beta)^2 - 4b\beta(2 - b)}}{2b(1 - \beta)}
\]  
(15)

The second order condition is satisfied since \( \frac{d^2\tilde{x}_{A}(t)}{dt^2} < 0 \) for all values of \( b \in (0, 1), \beta \in (0, 1) \) and \( t \in [0, 1) \). Moreover, the term within the square root in (15) is positive so that \( \tilde{t} \) is a real number. Note that if good 1 did not impose a negative externality, the optimal tax rate would be zero, i.e. \( \tilde{t}(b, \beta) \big|_{\beta=0} = 0 \). Table 1 shows \( \tilde{t}(b, \beta) \) for selected values of \( b \) and \( \beta \). Since \( \frac{\partial \tilde{t}}{\partial \beta} > 0 \) and \( \frac{\partial \tilde{t}}{\partial b} < 0 \), the following result holds.

**Lemma 2:** In a closed economy, the welfare-maximizing pollution tax rate \( \tilde{t}(b, \beta) \) increases as (i) the pollution damage parameter \( \beta \) increases, and (ii) the preference for the polluting good \( b \) decreases.

With optimal taxation in country A, the output of each good, \( \tilde{x}_{A}^{1} \big|_{t=\tilde{t}} \) and \( \tilde{x}_{A}^{2} \big|_{t=\tilde{t}} \), and the price ratio, \( \tilde{p}^{A} \big|_{t=\tilde{t}} \), are found by substituting (15) into (12) and (13).\(^{10}\) Given the utility function (4), consumers must consume positive amounts of both goods. Hence, complete specialization in the production of either good cannot be optimal under autarky. While imposition of the optimal tax improves country A’s welfare relative to a no-taxation scenario, country A still remains worse off compared to country B due to the pollution damage, i.e. \( \tilde{u}^{A} \big|_{t=0} < \tilde{u}^{A} \big|_{t=\tilde{t}} < \tilde{u}^{B} \). Further, comparison of the autarkic price ratios in the two countries leads to the following result.

**Proposition 1:** When country A imposes its welfare maximizing pollution tax \( \tilde{t}(b, \beta) \), its autarkic price ratio is higher than that in country B if and only if preference for the polluting good is sufficiently small. Specifically, \( \tilde{p}^{A} \big|_{t=\tilde{t}} \geq \tilde{p}^{B} = 1 \) if and only if \( b \leq \bar{b}(\beta) \), where \( \bar{b}(\beta) \) is implicitly given by the solution to \( \tilde{p}^{A} \big|_{t=\tilde{t}} = 1 \).

\(^{9}\)The larger of the two roots, which satisfy the FOC, is ignored as it leads to \( \tilde{t} > 1 \).

\(^{10}\)The autarky equilibrium when country A imposes its optimal tax is identical to the outcome where a social planner maximizes \( u^{A} \) subject to country A’s PPF \( q_{2}^{A} = (1 - q_{1}^{A}) (1 - \beta q_{1}^{A}) \). Substituting for \( q_{2}^{A} \), the planner’s maximization problem reduces to \( \max_{q_{1}^{A}} u^{A} = b \ln q_{1}^{A} + (1 - b) \ln (1 - q_{1}^{A}) (1 - \beta q_{1}^{A}) \). We find that \( \frac{d^2u^{A}}{dq_{1}^{A}^2} < 0 \) for \( b \in (0, 1), \beta \in (0, 1) \) and \( q_{1}^{A} \in (0, 1) \). Moreover solving the planner’s FOC, \( \frac{\partial u^{A}}{\partial q_{1}^{A}} = 0 \), we get \( q_{1}^{A} = x_{1}^{A} = \frac{1 + \beta - \sqrt{(1 + \beta)^2 - 4b\beta(2 - b)}}{2b(2 - b)} \), which is observed to be equal to \( \tilde{x}_{1}^{A} \big|_{t=\tilde{t}} \).
Proof: Using (13) and (15), we have \( \frac{d\wp^A(b, \beta, \tilde{u}(b, \beta))}{db} = \frac{\partial \wp^A}{\partial \bar{u}} + \frac{\partial \wp^A}{\partial \bar{t}} < 0 \), where \( \frac{\partial \wp^A}{\partial \bar{u}} > 0 \) and \( \frac{\partial \wp^A}{\partial \bar{t}} < 0 \) from Lemmas 1 and 2, and \( \frac{\partial \wp^A}{\partial \bar{u}} = -\frac{\beta}{(1 - b\beta)^2} < 0 \). For example, if \( \beta = 0.2 \), we have \( \tilde{p}_t |_{t=\bar{t}} \geq 1 \) if and only if \( b \leq 0.53 \).

In the absence of pollution taxation, country A has a comparative advantage in the polluting good vis-a-vis country B, as \( \tilde{p}_t |_{t=0} = 1 - b\beta < \tilde{p}_B \). By contrast, imposition of the optimal tax increases country A’s autarky price ratio such that, for \( b < \tilde{b}(\beta) \), it has a comparative advantage in the clean good. Such a policy-induced reversal of comparative advantage can enable a country with the (more severe) production externality problem to export the environmentally-sensitive good instead of the polluting good in the trading equilibrium. The reversal can also improve global productivity and the gains from trade, as we show below.

4 Small open economy

We begin by analyzing the trading equilibrium in a small open economy. Suppose both A and B are small countries that take the world price ratio, \( \hat{p} \equiv \frac{\tilde{p}_A}{\tilde{p}_B} \), as given when they trade with the rest of the world. In such a case, the pollution tax imposed by country A does not affect its terms of trade \( \hat{p} \). In our Ricardian model, a small country will specialize in the production of either good in the trading equilibrium, unless the world price ratio happens to be equal to its autarky price ratio. Below, we first specify the efficient pattern of specialization that maximizes country A and B’s welfare under trade, and then examine whether free trade would lead to this efficient pattern of specialization in these countries. A hat (\( ^\wedge \) ) over a variable denotes its value in the trading equilibrium.

If \( \hat{p} < 1 \), the efficient pattern of specialization under trade is one where both countries A and B specialize in good 2 and export it to the rest of the world. Each of these two countries then imports good 1 from the rest of the world, consumes \( \hat{x}^1_i = \frac{1}{p} \) and \( \hat{x}^2_i = 1 - b \), where \( i = A, B \), and obtains the welfare level of \( \hat{u}^i_I \equiv \hat{u}^B - \ln \hat{p} \hat{u}^B \). Conversely, if \( \hat{p} > 1 \), it is efficient for countries A and B to specialize in, and export, good 1 to the rest of the world. Each country then imports good 2, consumes \( \hat{x}^1_i = b \) and \( \hat{x}^2_i = \hat{p} (1 - b) \), and obtains the welfare level \( \hat{u}^i_{II} \equiv \hat{u}^B + (1 - b) \ln \hat{p} \hat{u}^B \). The above specialization patterns are efficient because \( \hat{u}^i_I \geq \hat{u}^i_{II} \) if and only if \( \hat{p} \leq 1 \). In other words, if \( \hat{p} < 1 \), each country is better off specializing in good 2 rather than good 1 (and vice versa for \( \hat{p} > 1 \)). Moreover, specializing in this efficient manner leads to positive gains from trade for both the countries, as their utility under trade exceeds their utility in the autarkic equilibrium. Note that, given our assumption of local pollution, the cross-sectoral externality problem in country A is
eliminated when it specializes in the production of either good. Hence, by enabling spatial separation of its mutually incompatible industries, the efficient pattern of trade provides additional gains for country A, which suffers from the cross-sectoral externality problem, over and above the conventional gains from trade obtained by country B.\footnote{We have \( \hat{\bar{u}}^A_{ij} - \bar{u}^A (t) > \hat{\bar{u}}^B_{ij} - \bar{u}^B = -b \ln \hat{p} > 0 \) if \( \hat{p} < 1 \), and \( \hat{\bar{u}}^A_{II} - \bar{u}^A (t) > \hat{\bar{u}}^B_{II} - \bar{u}^B = (1-b) \ln \hat{p} > 0 \) if \( \hat{p} > 1 \). Indeed, even if the world price ratio \( \hat{p} \) happened to be equal to country A’s autarky price ratio \( \hat{p}^{A} \), complete specialization in either good would lead to gains from trade for that country. Similar gains would not arise for country B, if the world price ratio were equal to its autarky price ratio, i.e. \( \hat{p} = \hat{p}^{B} \).}

Will the market-determined pattern of trade be efficient when the closed economies A and B open up to trade? Given its autarkic price ratio \( \hat{p}^{B} = 1 \), country B, which does not suffer from the externality problem, always specializes in the efficient manner when it opens up to trade. By contrast, in the absence of any regulation, country A’s autarkic prices may provide incorrect incentives to its producers regarding the direction of trade. Suppose, the parameters \( b, \beta \) and \( \hat{p} \) take values such that we have \( \hat{p}^{A}_{i=0} < \hat{p} < 1 \), where \( \hat{p}^{A}_{i=0} = 1 - b \beta \). In such a case, country B will export the clean good when it opens up to trade, as its autarkic price ratio \( \hat{p}^{B} \) exceeds the world price ratio \( \hat{p} \). However, since the polluting good is relatively cheaper in country A under autarky, it will export that good when trade opens up. This pattern of trade would be inefficient for country A as it would be better off specializing in and exporting the clean good rather than the polluting good (recall that \( \hat{\bar{u}}^i_I > \hat{\bar{u}}^i_{II} \) when \( \hat{p} < 1 \)).\footnote{Copeland and Taylor (1999, Proposition 3) showed that a small country would always gain from trade if it specializes in the efficient manner. However, they did not examine the possibility that autarky prices may provide an incorrect signal to a small country regarding the efficient pattern of specialization. Country A can lose from trade if it specializes in an inefficient manner (e.g. if it specializes in the polluting good, by subsidizing its production, when \( \hat{p} < 1 \)).} Imposition of a pollution tax by country A can correct this inefficiency by reversing the pattern of comparative advantage. By taxing its polluting sector under autarky, country A can raise its autarkic price ratio such that \( \hat{p}^{A} (t) > \hat{p} \). Then, in the ensuing trading equilibrium, the efficient pattern of specialization will result in country A will specialize in and export the clean good.

On the other hand, if the parameter values are such that \( \hat{p} < \hat{p}^{A}_{i=0} < 1 < \hat{p} \), the pattern of trade determined by autarkic prices without taxation in country A is efficient. If \( \hat{p} < \hat{p}^{A}_{i=0} < 1 \), country A specializes in and exports the clean good, whereas if \( \hat{p}^{A}_{i=0} < 1 < \hat{p} \), it specializes in and exports the polluting good.\footnote{In the trading equilibrium, for country A to specialize in the production of the clean (respectively, polluting) good, its pollution tax rate \( \hat{t} \) should exceed (respectively, be less than) \( \frac{\hat{p}^{A} - 1}{\hat{t}} \). Then, production of the polluting (respectively, clean) good would be unprofitable, while production of the clean (respectively, polluting) good yields zero profit.}
5 Trade in a two-country world

We now turn to examine the equilibrium where A and B are large countries that trade with each other, and the world price ratio $\hat{p}$ is determined endogenously. Given the Ricardian structure of our model, the following mutually-exclusive patterns of specialization can potentially arise in the trading equilibrium: (i) country $i$ specializes in the clean good while the other country diversifies (i.e. produces both goods), (ii) country $i$ specializes in the clean good while the other country specializes in the polluting good, and (iii) country $i$ specializes in the polluting good while the other country diversifies. Since $i = A, B$, the aforementioned describes six alternative patterns of specialization. In three of these six cases, country A exports the polluting good while, in the other three cases, it exports the clean good.

To derive the trading equilibrium, we consider each of the six possible patterns of specialization, and examine whether that specialization pattern can be a feasible equilibrium for some combination of parameter values. In what follows, we report the trading equilibrium for two alternative scenarios. In the benchmark scenario, country A does not pursue an environmental policy while, in the second scenario, country A imposes a pollution tax at a rate that maximizes its welfare in the trading equilibrium. In each scenario, country B does not impose any tax.

5.1 Equilibrium without taxation: The benchmark

In this subsection, we consider the equilibrium when country A does not regulate its polluting sector. This benchmark scenario is similar to the trading equilibrium in Copeland and Taylor (1999), who used a dynamic model to consider inter-temporal changes in the stock of pollution. The equilibriums of our simpler static model replicate their steady state solutions and facilitate the comparison of equilibriums across scenarios. We use our benchmark scenario to examine how taxation of its polluting sector by country A alters the gains from trade for each country relative to the benchmark.

Proposition 2 identifies the trading equilibrium associated with different values of the preference parameter $b$.

**Proposition 2**: The trading equilibrium in the absence of pollution taxation by country $A$ is as follows:

(i) If $b \in \left(0, \frac{1-\beta}{2+\beta}\right)$, country A is diversified while country B is specialized in good 2. Equilibrium welfare in the two countries are $\hat{u}^A = \hat{u}^B + (1 - b) \ln \hat{p}$ and $\hat{u}^B = \hat{u}^B - b \ln \hat{p}$, where $\hat{p} \equiv \hat{p}_1 = \frac{1}{2} \left(1 - b/\beta + \sqrt{1 - 6b/\beta^2 + b^2/\beta^2}\right) < 1$ is the world relative price of good 1.
(ii) If \( b \in \left( \frac{1-\beta}{2+\beta}, \frac{1}{2} \right) \), country A is specialized in good 1 and country B is specialized in good 2. Equilibrium welfare in the two countries are \( \bar{w}^A = b \ln b + (1 - b) \ln b \) and \( \bar{w}^B = b \ln (1 - b) + (1 - b) \ln (1 - b) \). Moreover, \( \bar{p} = \frac{b}{1-b} < 1 \) in this equilibrium.

(iii) If \( b \in \left( \frac{1}{2}, 1 \right) \), country A is specialized in good 1 and country B is diversified. Equilibrium welfare in the two countries are \( \bar{w}^A = \bar{w}^B = \bar{w}^A \), and the world price ratio is \( \bar{p} = 1 \).

**Proof:** Omitted. See appendix A in Benaroch and Thille (2001) for a similar proof.\(^{14}\)

In the presence of cross-sectoral externalities, the spatial rearrangement of production that takes place as a result of trade affects global productivity, though not necessarily in a positive manner. Global productivity improves if country A is specialized in the trading equilibrium, and worsens if trade leads to diversification by country A.\(^{15}\) Given our production functions and the total supply of labour, when country A specializes in the production of either good, the global production possibilities frontier is given by \( q_2 = 2 - q_1 \), where \( q_j \equiv q_j^A + q_j^B \) is the global output of good \( j = 1, 2 \). On the other hand, when country A diversifies in the trading equilibrium, the global PPF is given by\(^{16}\)

\[
q_2 = \begin{cases} 
1 + (1 - q_1) (1 - \beta q_1) & \text{if } 0 \leq q_1 \leq 1 \\
(1 - (q_1 - 1)) (1 - \beta (q_1 - 1)) & \text{if } 1 \leq q_1 \leq 2
\end{cases}
\]

(16)

Thus, switching from the efficient (where A is specialized) to the inefficient (where A is diversified) pattern of specialization has the effect of lowering the global PPF – from the solid straight line to the dotted curved line in Figure 1. Proposition 2 shows that free trade leads to an inefficient pattern of specialization if and only if \( b < \frac{1-\beta}{2+\beta} \).

The change in global productivity due to the spatial rearrangement of production modifies the conventional gains from trade that arise when two countries, with different opportunity costs of producing goods, trade with each other. Moreover, the distribution of the

\(^{14}\)None of the remaining three of the aforementioned six patterns of specialization can be an equilibrium, when country A does not impose a tax. For example, the pattern of specialization where A specializes in good 2 and B specializes in good 1 cannot be an equilibrium as this would require \( \frac{w^A}{p_1} > 1 \), \( \frac{w^A}{p_2} = 1 \), \( \frac{w^B}{p_1} = 1 \) and \( \frac{w^B}{p_2} > 1 \). The real wages in A imply the world price ratio is \( \frac{p^B}{p^A} < 1 \), while the real wages in B imply \( \frac{p^B}{p^A} > 1 \), which are contradictory. In a similar manner, the other patterns of specialization can be ruled out as feasible equilibriums for any set of parameter values. Note that, when A specializes in good 1 in the benchmark scenario, its real wages are \( \frac{w^A}{p_1} = 1 \) and \( \frac{w^A}{p_2} > 1 - \beta \).

\(^{15}\)Suppose, in the trading equilibrium, country A diversifies and country B specializes in the production of either good. This pattern of production is globally inefficient as more of good 2 could be produced for the same amount of good 1 by reversing this pattern of production, so that country A specializes and country B diversifies.

\(^{16}\)Suppose A diversifies in the trading equilibrium. If \( q_1 \in [0, 1] \), we have \( q_2^A = 1 \), \( q_1 = q_1^A \) and \( q_1 = (1 - q_1^A) (1 - \beta q_1^A) \), so that \( q_2 = q_2^A + q_2^B = 1 + (1 - q_1) (1 - \beta q_1) \). Alternatively if \( q_1 \in [1, 2] \), we have \( q_1^B = 1 \), \( q_1^A = q_1 - 1 \), and \( q_2 = q_2^A = (1 - q_1^A) (1 - \beta q_1^A) \).
global gains from trade between the countries depends on the terms of trade \( \tilde{p} \) that gets established in a particular trading equilibrium. Specifically, we find that in the trading equilibrium described in Proposition 2, country A, which exports the polluting good, loses from trade (i.e. \( \tilde{u}^A \leq \tilde{u}^A\big|_{t=0} \)) if and only if \( b \leq b^L \), where

\[
\frac{1 - \beta}{2 - \beta} < b^L \equiv \frac{1}{2\beta} \left( \beta + 2 - \sqrt{\beta^2 + 4} \right) < \frac{1}{2}
\]  

(17)

Thus, country A always loses from trade if \( b \in \left( 0, \frac{1 - \beta}{2 - \beta} \right) \), and may lose from trade if \( b \in \left( \frac{1 - \beta}{2 - \beta}, \frac{1}{2} \right) \). By contrast, trade never leads to losses for country B, which exports good 2. When \( b \in \left( 0, \frac{1}{2} \right) \), this country gains from trade (i.e. \( \tilde{u}^B > \tilde{u}^B \)), while trade leaves country B’s welfare unchanged from its autarkic level when \( b \in \left( \frac{1}{2}, 1 \right) \).

Turning to global gains from trade, defined as the sum of the two countries’ gains from trade, we find that if \( b \leq 0.32 \) the global gains are negative for all values of \( \beta \in (0, 1) \). Thus, when country A diversifies in the trading equilibrium, global welfare under trade can be lower than that under autarky. By contrast, when country B diversifies in the trading equilibrium, global gains from trade are positive. These benchmark results are in line with Copeland and Taylor (1999) who found that, for identical countries, the exporter of the polluting good loses from trade for sufficiently small values of \( b \), while the exporter of the clean good always gains from trade.

### 5.2 Equilibrium with taxation by country A

We now analyze the trading equilibrium which allows for the possibility that country A imposes an environmental tax on domestic producers of the polluting good. As before, to derive this equilibrium, we consider all possible patterns of specialization, where country A exports the polluting good and alternatively where it imports the polluting good, and identify the range of parameter values and tax rate for which each specialization pattern can be an equilibrium. Moreover, when alternative specialization patterns are feasible as equilibrium for any given values of \( b \) and \( \beta \), we select the equilibrium that maximizes country A’s welfare as the one that would be chosen by country A when it non-cooperatively sets its tax rate.\(^\text{18}\) Proposition 3 identifies the trading equilibrium in the presence of environmental taxation.

\(^{17}\text{To compute the gains from trade for country A in the benchmark scenario, we use its utility in the autarkic equilibrium without taxation, i.e. } \tilde{u}^A\big|_{t=0}.\)

\(^{18}\text{See the Appendix for additional details on the derivation of this trading equilibrium. Given our Ricardian model and a non-convex production possibility set, welfare under trade becomes a discontinuous function of } t. \text{ Discrete jumps in country A’s welfare level take place as the equilibrium changes from one specialization pattern to another. Hence, a marginal analysis is insufficient for deriving country A’s optimal tax rate.}\)
taxation by country A.

**Proposition 3:** The trading equilibrium when country A sets a welfare-maximizing pollution tax is as follows:

(i) If $b \in (0, \frac{1}{2})$, country A is specialized in good 2 and country B is diversified. In this equilibrium, the tax rate is sufficiently high, i.e. 
\[ \hat{t} \geq \frac{1+b\beta}{b(1+\beta)} \equiv \overline{t}. \]

(ii) If $b \in (\frac{1}{2}, 1)$, country A is specialized in good 1 and country B is diversified. In this equilibrium, the tax rate is sufficiently low, i.e. 
\[ \hat{t} \leq \min \{ \beta, \frac{2b-1}{b}, t^L \} \text{ where } t^L \equiv \frac{1}{2b}(1+b\beta - \sqrt{(1+b\beta)^2 - 4b\beta (2b-1)}) > 0. \]

In each of these equilibriums, welfare in the two countries are \( \overline{w}^A = \overline{w}^B = \overline{w}^B \), and the world price ratio is \( \overline{p} = 1. \)

**Proof:** See the Appendix.

As shown in the Appendix, if the polluting good is less preferred than the clean good, i.e. $b < \frac{1}{2}$, country A obtains a higher level of welfare by specializing in the clean good and exporting it to country B, than by choosing an alternative pattern of trade. In order to ensure that this specialization pattern transpires, country A sets a sufficiently high pollution tax, \( \hat{t} \geq \overline{t} \), so as to make production of the polluting good unprofitable domestically. Since the polluting good is not produced in country A, it does not collect any pollution tax revenue in this equilibrium.

When $b < \frac{1}{2}$, two alternative specialization patterns that are feasible as equilibrium are those where “A is diversified and B specializes in good 2”, and “A specializes in good 1 and B specializes in good 2” (cases 2 and 3 in the Appendix). In these cases, country A exports the polluting good, and has to set its tax at a rate lower than \( \overline{t} \). However, these alternative patterns of specialization leave country A with lower levels of welfare compared to specializing in the clean good. As mentioned, we assume that country A would not choose such tax rates that make it worse-off. In particular, we find that in the equilibrium where country A diversifies and exports the polluting good (case 2), an increase in the pollution tax rate raises the world relative price of the polluting good, which improves country A’s terms of trade and welfare. Hence, country A has an incentive to raise its tax on the polluting sector in this case. This situation constitutes a reversal of the usual pollution haven result, where countries reduce their environmental taxes in an effort to gain a competitive advantage over their trading partners. In fact, when $b < \frac{1}{2}$, we find that country A obtains the highest level of welfare by raising its tax all the way to \( \overline{t} \), so that it specializes in the production of the clean good.

\*tax in the trading equilibrium.
On the other hand, if the polluting good is more preferred than the clean good, i.e. \( b > \frac{1}{2} \), the welfare-maximizing equilibrium for country A is to specialize in and export the polluting good. For this to happen, country A must set a sufficiently low rate of tax, i.e. \( \tilde{t} < \min \{ \beta, \frac{2b-1}{b}, t^L \} \). The environmental tax can therefore be positive or even zero in this equilibrium. When \( b > \frac{1}{2} \), the only other feasible equilibriums are those where country A diversifies or specializes in good 2, while country B specializes in good 1 (cases 5 and 6 in the Appendix). In these cases, country A must set the tax at a higher level so that it exports the clean good. However, these alternative patterns of specialization leave country A with lower levels of welfare relative to that when it specializes in the polluting good. In particular, we find that in the equilibrium where country A diversifies and imports the polluting good (case 5), reducing the tax lowers the relative price of the polluting good and improves country A’s terms of trade and welfare.

Thus, in the trading equilibrium, the pollution tax rate chosen by country A not only accounts for the production externality imposed by its polluting sector but also the associated change in the terms of trade for country A. While the tax plays a crucial role in equilibrium selection for country A, Proposition 3 shows that the world price ratio and welfare of each country in the welfare-maximizing equilibrium are independent of (marginal changes in) the environmental tax. Comparing utility levels in the autarkic and trading equilibriums, we find that country A always gains from trade when it is able to tax, as \( \tilde{u}^A = \tilde{u}^B > \tilde{u}^A(t) \). On the other hand, country B’s welfare remains unchanged with trade. Global gains from trade are, therefore, always positive. Since country A is not diversified, the pattern of specialization described in Proposition 3 is globally efficient for all values of \( b \).

5.3 Discussion

A comparison of the trading equilibriums given by Propositions 2 and 3 shows that when country A imposes an environmental tax on its polluting sector, welfare can rise both in country A as well as globally. In particular, we find that country A is better off by choosing the tax such that it completely specializes in the good that is more preferred by consumers. As a result, it can eliminate the impact of pollution on productivity and benefit from more favourable terms of trade.

If \( b \leq b^L \), taxation enables country A to switch from exporting the dirty good and losing from trade (in the absence of taxation) to exporting the clean good and gaining from trade. Moreover, if \( b < \frac{1}{2} \), country A’s welfare in the trading equilibrium with taxation exceeds its welfare level in the trading equilibrium without taxation. Thus, for sufficiently small values
of the preference parameter $b$, taxation changes the trading equilibrium from an inefficient to an efficient pattern of specialization, improves global productivity, and converts trade from being globally welfare-deteriorating to welfare-improving. On the other hand, if $b > \frac{1}{2}$, the trading equilibriums with and without environmental taxation are identical (where country A is specialized in good 1 and country B is diversified). If preference for the polluting good is large, the pattern of trade is efficient even in the absence of taxation and neither country loses from trade. Imposition of a tax by country A does not lead to additional welfare gains in such a situation.

Recall that, in the autarkic equilibrium without taxation, country A has a comparative advantage in the polluting good over country B. If the direction of trade is determined by these autarky price ratios, then country A will export the polluting good to country B when they trade with each other. However, if preference for the polluting good is small, this pattern of trade would be globally inefficient and welfare-deteriorating for country A. Imposition of a pollution tax under autarky and reversing the direction of trade would then enable country A to correct this inefficiency and gain from trade.

Note that the above results are obtained under the stark assumption that only one country suffers from cross-sectoral production externalities. However, the beneficial effects of pollution taxation by country A arise not only when it solely suffers from such externalities, but also when both the countries do so. If the domestic polluting sector adversely, but differentially, affects the environmentally sensitive sector in each country, global productivity worsens when the country with the higher value of the pollution damage parameter diversifies in the trading equilibrium. The next section numerically analyzes this more realistic situation. It shows that free trade can again lead to an inefficient pattern of specialization, which can be corrected by unilateral taxation in the dirtier country.

6 Numerical analysis

The model analyzed so far assumed, for expositional simplicity, that country B does not suffer from any cross-sectoral externality. In this section, we relax that assumption by modifying equation (3) as follows:

$$q_2^B = (1 - \gamma q_1^B) L_2^B,$$

where $\gamma \in (0, 1)$ is the extent of the negative externality imposed by sector 1 on sector 2 in country B. While the previous sections assumed $\gamma = 0$, this section allows for pollution damage in both countries. Nevertheless, we continue to assume that the externality problem
is less pronounced in country B than it is in country A, and in particular that

\[ \gamma = 0.1 < \beta = 0.2 \]

All other aspects of our previous analysis remain unchanged in this analysis, including the assumption that country B does not impose any tax. Given (18), the real wage in country B now satisfies \( \frac{w^B}{p^*_1} = 1 \) and \( \frac{w^B}{p^*_2} = 1 - \gamma q^B_1 \), when both the goods are produced in that country.

In the autarkic equilibrium, when there is no regulation, the higher pollution damage in country A causes the relative price of the polluting good to be lower in that country, i.e. \( \tilde{p}^A|_{t=0} = 1 - b\beta < \tilde{p}^B = 1 - b\gamma \). In other words, the dirtier country A has a comparative advantage in the polluting good. By contrast, imposition of the optimal pollution tax under autarky, given by (15), in country A increases the relative price of the polluting good in that country such that \( \tilde{p}^A|_{t=1} \geq \tilde{p}^B \) if and only if \( b \leq 0.69 \).

In the absence of any regulation, the trading equilibrium is as follows: (i) if \( b \in (0, 0.45) \), country A is diversified while country B is specialized in good 2; (ii) if \( b \in (0.45, 0.5) \), country A is specialized in good 1 and country B is specialized in good 2; and (iii) if \( b \in (0.5, 1) \), country A is specialized in good 1 and country B is diversified. Thus, when \( b < 0.45 \), the market determined pattern of trade is inefficient, as country A, with the more severe pollution damage, diversifies. We find that country A, which exports the polluting good, loses from trade if \( b < 0.48 \), but gains for higher values of \( b \). Country B, on the other hand, exports the clean good and always gains from trade. Moreover, global gains from trade are negative if and only if \( b < 0.24 \).

By contrast, with environmental taxation by country A, the trading equilibrium is as follows: (i) if \( b \in (0, 0.47) \), country A is specialized in good 2 while country B is diversified; (ii) if \( b \in (0.47, 0.5) \), country A is specialized in good 2 and country B is specialized in good 1; (iii) if \( b \in (0.5, 1) \), country A is specialized in good 1 while country B is diversified. This trading equilibrium maximizes country A’s welfare, and is derived in a manner similar to the derivation of Proposition 3 described in the Appendix. If \( b < 0.5 \), country A specializes in and exports the clean good, and imposes a sufficiently high tax, specifically \( \hat{t} > \frac{9b - 4}{5b} \), so as to discourage domestic production of the polluting good. Similar to the results derived in Section 5, when preference for the polluting good is small, country A has an incentive to drive out the polluting sector and let it reduce productivity of the environmentally sensitive fluids.

\[ ^{19} \text{When } \gamma > 0 \text{, the variables such as production, consumption and welfare in the trading equilibria involve large expressions in } b, \beta \text{ and } \gamma \text{. Hence, for expositional ease, we assume specific values of } \beta \text{ and } \gamma \text{ in this section. We considered other values for } \beta \text{ and } \gamma \text{ such that } \beta > \gamma \text{, and found that alternative specifications of the damage parameters yield qualitatively similar results.} \]
industry elsewhere. Conversely, if \( b > 0.5 \), country A specializes in the polluting good, and the tax must be sufficiently small, i.e. \( \hat{t} < \min\{\frac{2b-1}{b}, \frac{2b-3}{2b-1}\} \).\footnote{Note that, \( \frac{2b-3}{2b-11} < \frac{2b-1}{b} \) if and only if \( b > 0.55 \).} In the trading equilibrium with taxation, the pattern of specialization is efficient for all values of \( b \). We find that country A always gains from trade, as its utility in the trading equilibrium exceeds its utility in the autarkic equilibrium with the optimal tax, i.e. \( \hat{u}^A > \hat{u}^A_{|t=t^*} \). On the other hand, trade leads to losses for country B if and only if \( b < 0.49 \). Global gains from trade are, however, positive for all values of \( b \). The numerical analysis thus confirms the analytical results derived in the previous sections of the paper.

7 Conclusion

The paper analyzes the effects of unilateral pollution taxation on the pattern of and the gains from trade, when a polluting sector negatively affects productivity in another, environmentally sensitive, sector. We use a general equilibrium framework where two countries differ in terms of their pollution damage parameter and the dirtier country, with the more severe production externality problem, imposes a tax on production of the polluting good. Our results show that, by increasing the autarky relative price of the polluting good, the environmental tax can enable the dirtier country, irrespective of whether it is small or large, to reverse its comparative advantage and move from an inefficient to an efficient pattern of trade. This can not only benefit the dirtier country but also increase global productivity and the global gains from trade. Our results suggest that, when polluting goods have a lower preference or price than environmentally sensitive goods, countries with significant production externality problems ought to go for it alone and impose unilateral environmental taxes even when their trading partners, with less severe production externality problems, do not do so.

Our numerical analysis shows that when large countries trade with each other, and the dirtier country imposes an environmental tax, the cleaner country can end up exporting the polluting good and losing from trade. This leads to the possibility that the cleaner country may want to tax its polluting sector as well, so as to obtain a competitive advantage in the clean good. Taxation by both countries is likely to lead to an equilibrium similar to the no-taxation scenario, where the pattern of trade is inefficient and the global PPF is lower. In such a case, the globally efficient outcome may require that the dirtier country impose a unilateral tax and offer side payments to the cleaner country for not taxing its polluting sector. A formal analysis of pollution taxation by both countries in the presence
of cross-sectoral externalities is left as future work.

Appendix

Proof of Proposition 3: In this appendix, we derive the equilibrium when countries A and B trade with each other, and country A can impose a tax on domestic producers of good 1. As mentioned, we consider all possible patterns of specialization, where A exports the polluting good and alternatively where it imports that good, and derive the parameter values and the tax rate for which each specialization pattern can be an equilibrium. We also compute welfare of the two countries in each equilibrium. Finally, for any given value of $b$ and $\beta$, we select the equilibrium that maximizes country A’s welfare as the one that would be chosen by country A when it sets its tax rate.

Six mutually-exclusive patterns of specialization that can potentially arise under trade are analyzed below (cases 1-6). We find that a necessary condition for the first three of these to be an equilibrium is $b < \frac{1}{2}$, while the last three patterns of specialization can be an equilibrium only if $b > \frac{1}{2}$.

(i) **Case 1:** A specializes in good 2 and B is diversified. In this case, Country A uses its entire labour for producing the clean good, i.e. $q_1^A = 0$ and $q_2^A = 1$. Since B is diversified, we have $q_j^B \in (0, 1)$, where $j = 1, 2$. This production pattern implies that real wages in A must satisfy $\frac{w^A}{p_1} > 1 - t$ and $\frac{w^A}{p_2} = 1$. Moreover, as both the goods are produced in country B, we have $\frac{w^B}{p_1} = 1$ and $\frac{w^B}{p_2} = 1$, which implies that the world price ratio is $p \equiv \frac{p_1}{p_2} = 1$. Consequently $\frac{w^A}{p_1} > 1 - t$ is satisfied as long as $t > 0$.

Substituting the real wages into equations (5) and (11) gives the consumption of the two goods in each country as $x_1^A = b \left( \frac{w^A}{p_1} + tq_1^A \right) = b$, $x_2^A = p \left( 1 - b \right) \left( \frac{w^A}{p_1} + tq_1^A \right) = 1 - b$, $x_1^B = \frac{bw^B}{p_1} = b$, and $x_2^B = \frac{(1-b)w^B}{p_2} = 1 - b$. In this equilibrium, country A exports the clean good to, and imports the dirty good from, country B. Equating world demand and supply for each good, we find that $q_1^B = x_1^A + x_1^B - q_1^A = 2b$, and $q_2^B = x_2^A + x_2^B - q_2^A = 1 - 2b$. Since $q_2^B > 0$ when B is diversified, a necessary condition for this pattern of specialization to be an equilibrium is $b < \frac{1}{2}$.

As their consumption levels are the same, the two countries obtain equal welfare in this equilibrium, given by $u^A_{case 1} (b) = u^B_{case 1} (b) = b \ln b + (1 - b) \ln (1 - b) = \tilde{u}^B$.

(ii) **Case 2:** A is diversified and B specializes in good 2. In this case we have $q_j^A \in (0, 1)$, $j = 1, 2$, which implies real wages in country A are $\frac{w^A}{p_1} = 1 - t$ and $\frac{w^A}{p_2} = 1 - \beta q_1^A$. Moreover,
$q^B_1 = 0$ and $q^B_2 = 1$, and real wages in country B satisfy $\frac{w^B_0}{p_1} > 1$ and $\frac{w^B_0}{p_2} = 1$. These real wages in A and B, in turn, imply that the world price ratio is given by $p = \frac{1 - \beta q^A_1}{1-t} < 1$.

Substituting the real wages into (5) and (11) gives the consumption of the two goods in each country as $x^A_1 = b \left(1 - t + t q^A_1\right)$, $x^A_2 = p \left(1 - b\right) \left(1 - t + t q^A_1\right)$, $x^B_1 = \frac{b}{p}$, and $x^B_2 = 1 - b$. In this equilibrium, country A exports the dirty good and imports the clean good. Equating world demand and supply for each good and solving, we find that $q^A_1 = x^A_1 + x^B_1 - q^B_1 = \frac{b(1+p-p\mu t)}{p(1-b)t}$, and $q^A_2 = x^A_2 + x^B_2 - q^B_2 = \frac{(1-t)(p-b-bp)}{1-bt}$. As $q^A_2 > 0$, we have $p > \frac{b}{1-b}$ in this case. The latter, together with the above-mentioned condition $p < 1$, implies that a necessary condition for this equilibrium is $b < \frac{1}{2}$. Solving $p = \frac{1 - \beta q^A_1}{1-t}$ and $q^A_1 = \frac{b(1+p-p\mu t)}{p(1-bt)}$ simultaneously, we get

$$q^A_1(t) = \frac{1 + b\beta - bt(1 + \beta) - \sqrt{G}}{2 \beta (1 - bt)}$$

where $G \equiv (1 + b^2\beta^2 - 6b\beta + b^2t^2 - 3b\beta - 3\beta + 1)$. By substituting for $q^A_1(t)$ appropriately, we obtain the world price ratio and welfare of country A as $p(b, \beta, t)$ and $u^A_{case\, 2}(b, \beta, t)$, respectively.\(^{21}\)

Numerical simulations show that $\frac{\partial q^A_1(t)}{\partial t} < 0$, $\frac{\partial p(b, \beta, t)}{\partial t} > 0$, $\frac{\partial}{\partial t} \left(u^A_{case\, 2}(b, \beta, t)\right) > 0$ and $u^A_{case\, 2}(b, \beta, t) < u^A_{case\, 1}(b)$ for all values of $b \in \left(0, \frac{1}{2}\right)$, $\beta \in (0, 1)$ and $t \in [0, 1)$ in this equilibrium. Moreover, when $t = \frac{1+b\beta}{b(1+\beta)} \equiv \tilde{t}$, we have $G = \frac{8b^2(1-b)^2}{(\beta+1)^2}$ and $q^A_1(t) \mid_{t=\tilde{t}} < 0$. Thus, the tax imposed by country A has to be sufficiently less than $\tilde{t}$ for production of the dirty good to take place in that country.

Since country A exports the dirty good in this equilibrium, an increase in the tax improves its terms-of-trade by raising the relative price of the dirty good. However, even though $u^A_{case\, 2}(b, \beta, t)$ increases with $t$, it is always less than $u^A_{case\, 1}(b)$. Consequently, country A has an incentive to raise its tax to $\tilde{t}$, drive out the dirty industry to the other country, and specialize in the clean good (i.e. move from case 2 to case 1).

(iii) Case 3: A specializes in good 1 and B specializes in good 2. In this case we have $q^A_1 = 1$, $q^A_2 = 0$, $q^B_1 = 0$ and $q^B_2 = 1$, which implies real wages are $\frac{w^A_0}{p_1} = 1 - t$, $\frac{w^A_0}{p_2} > 1 - \beta$, $\frac{w^B_0}{p_1} > 1$ and $\frac{w^B_0}{p_2} = 1$. These real wages in A and B, in turn, imply that the world price ratio satisfies $\frac{1 - \beta}{1-t} < p < 1$.

Substituting the real wages into (5) and (11) gives the consumption of the two goods in each country as $x^A_1 = b$, $x^A_2 = p(1 - b)$, $x^B_1 = \frac{b}{p}$ and $x^B_2 = 1 - b$. Country A exports the dirty good and imports the clean good. Equating world demand and supply for each good,\(^{21}\)

Note that, in the absence of taxation by country A, we have $q^A_1(t) \mid_{t=0} = \frac{1+b\beta}{\sqrt{1+b^2\beta^2-6b\beta}}$ and $p(b, \beta, t) \mid_{t=0} = \frac{1 - \beta q^A_1}{1-t} \mid_{t=0} = \frac{1}{2} \left(1 - b\beta + \sqrt{1 - 6b\beta + b^2\beta^2}\right)$. The latter is identical to the value of the world price ratio under this pattern of specialization in Proposition 2(i), as expected.
\[ x_1^A + x_1^B = q_1^A \text{ and } x_2^A + x_2^B = q_2^B, \] we find that \( p = \frac{b}{1-b} \) in this equilibrium. The above-mentioned condition \( \frac{1\!-\!\beta}{1-t} < p < 1 \) is satisfied when \( t < \frac{b(2-\beta)-(1-\beta)}{b} \equiv \tilde{t} \) and \( \frac{1\!-\!\beta}{2-\beta} < b < \frac{1}{2} \). Note that \( \tilde{t} - \tilde{t} = \frac{(1-b)(2-\beta^2)}{b(\beta+1)} > 0 \).

Substituting \( p = \frac{b}{1-b} \) in the above equations specifying consumptions, we obtain the equilibrium welfare of each country as \( u_{\text{case } 3}^A (b) = b \ln b + (1-b) \ln b \) and \( u_{\text{case } 3}^B (b) = b \ln (1-b) + (1-b) \ln (1-b) \). For \( b < \frac{1}{2} \), we have \( u_{\text{case } 3}^A (b) < u_{\text{case } 1}^A (b) \).

(iv) **Case 4**: A specializes in good 1 and B is diversified. Now we have \( q_1^A = 1, q_2^A = 0 \), and \( q_1^B \in (0, 1) \), which implies real wages satisfy \( \frac{w^A}{p_1} = 1-t, \frac{w^A}{p_2} > 1-\beta, \frac{w^B}{p_1} = 1 \) and \( \frac{w^B}{p_2} = 1 \). Then the world price ratio is \( p = 1 > \frac{1\!-\!\beta}{1-t} \). The latter implies \( t < \beta \), for this pattern of specialization to be an equilibrium.

Substituting these real wages into (5) and (11) gives the consumption of the two goods in each country as \( x_1^A = b, x_2^A = 1-b, x_1^B = b \) and \( x_2^B = 1-b \). Country A exports the dirty good and imports the clean good. Equating world demand and supply for each good, we have \( q_1^B = x_1^A + x_1^B - q_1^A = 2b - 1 \) and \( q_2^B = x_2^A + x_2^B = 2(1-b) \). Since \( q_1^B > 0 \), we have \( b > \frac{1}{2} \) is a necessary condition for this equilibrium.

Equilibrium welfare of each country is \( u_{\text{case } 4}^A (b) = u_{\text{case } 4}^B (b) = b \ln b + (1-b) \ln (1-b) = \tilde{u}^B \).

(v) **Case 5**: A is diversified and B specializes in good 1. Now we have \( q_1^B \in (0, 1), q_1^A = 1 \) and \( q_2^B = 0 \), which implies the real wages are \( \frac{w^A}{p_1} = 1-t, \frac{w^A}{p_2} = 1-\beta q_1^A, \frac{w^B}{p_1} = 1 \) and \( \frac{w^B}{p_2} > 1 \). These real wages in A and B imply that the world price ratio satisfies \( p = \frac{1\!-\!\beta q_1^A}{1-t} > 1 \).

Substituting the real wages into (5) and (11) gives the consumption of the two goods in each country as \( x_1^A = b \left(1-t+qt^A\right), x_2^A = p \left(1-b\right) \left(1-t+qt^A\right), x_1^B = b \) and \( x_2^B = p \left(1-b\right) \). Further, equating world demand and supply for each good and solving, we find that \( q_1^A = x_1^A + x_1^B - q_1^A = \frac{2b-1}{1-bt} \) and \( q_2^A = x_2^A + x_2^B - q_2^B = \frac{2p(1-b)(1-t)}{1-bt} \). As \( q_1^A > 0 \), we have \( t < \frac{2b-1}{b} \) and \( b > \frac{1}{2} \) as necessary conditions for this equilibrium. Further, by substituting \( q_1^A = \frac{2b-1}{1-bt} \), we obtain \( p = \frac{1-q_1^A}{1-t} = \frac{1-\beta (2b-1)-bt (1-\beta)}{(1-t)(1-bt)} \), and \( p > 1 \) if and only if \( t \left(1+b\beta\right) - \beta \left(2b-1\right) - bt^2 > 0 \). The latter condition is satisfied if and only if \( t \in (t^L, t^H) \), where

\[
t^L, t^H \equiv \frac{1}{2b} \left(1+b\beta \pm \sqrt{(1+b\beta)^2 - 4b\beta (2b-1)}\right)
\]

Numerical simulations show that \( \frac{\partial u^A}{\partial t} > 0, \frac{\partial u^A}{\partial t} \left(u^A_{\text{case } 5} (b, \beta, t)\right) < 0, \) and \( u^A_{\text{case } 5} (b, \beta, t) < u^A_{\text{case } 4} (b) \) for all values of \( b \in \left(\frac{1}{2}, 1\right), \beta \in (0, 1) \) and \( t \in [0, 1) \) in this equilibrium. Since country A imports the dirty good, an increase in the tax worsens its terms-of-trade and welfare. Further, since \( u^A_{\text{case } 5} (b, \beta, t) \) is less than \( u^A_{\text{case } 4} (b) \), country A has an incentive to
lower its tax below \( t^L \) so that it can specialize in the polluting good rather than diversify (i.e. move from case 5 to case 4).

(vi) Case 6: A specializes in good 2 and B specializes in good 1. In this case we have \( q_1^A = 0 \), \( q_2^A = 1 \), \( q_1^B = 1 \) and \( q_2^B = 0 \). This pattern of production implies that real wages in countries A and B satisfy the following conditions: \( \frac{w^A}{p_1} > 1 - t \), \( \frac{w^A}{p_2} = 1 \), \( \frac{w^B}{p_1} = 1 \) and \( \frac{w^B}{p_2} > 1 \). These real wages in A and B, in turn, imply that the world price ratio satisfies \( 1 < p < \frac{1}{1-t} \).

Substituting the real wages in (5) and (11) gives the consumption of the two goods in each country as \( x_1^A = \frac{b}{p} \), \( x_2^A = 1 - b \), \( x_1^B = b \), and \( x_2^B = p(1-b) \). Equating world demand and supply for each good, \( x_1^A + x_1^B = q_1^B \) and \( x_2^A + x_2^B = q_2^A \), we find that \( p = \frac{b}{1-b} \) in this equilibrium. The above-mentioned condition \( 1 < p < \frac{1}{1-t} \) is satisfied when \( b > \frac{1}{2} \) and \( t > \frac{b-1}{b} \).

Equilibrium welfare levels of the two countries in this case are given by \( u_{case\ 6}^A (b) = b \ln (1 - b) + (1 - b) \ln (1 - b) \) and \( u_{case\ 6}^B (b) = b \ln b + (1 - b) \ln b \). For \( b > \frac{1}{2} \), we have \( u_{case\ 6}^A (b) < u_{case\ 4}^A (b) \).

Using the above results, we can now identify the trading equilibrium that would be chosen by country A when it sets its tax rate to maximize welfare. If demand for the polluting good is small, i.e. \( b < \frac{1}{2} \), country A will set \( t \geq \bar{t} = \frac{1+\beta \bar{b}}{b(1+\beta)} \) and specialize in good 2 while country B diversifies (case 1). Thus, the tax rate has to be sufficiently high so as to make production of the polluting good unprofitable in country A. This gives country A the welfare level \( u_{case\ 1}^A (b) \), which exceeds the welfare it could get from the alternative patterns of specialization (cases 2 and 3). Recall that \( \bar{t} > \bar{t} \), and if \( t \geq \bar{t} \) (respectively, \( t > \bar{t} \) ) case 2 (respectively, case 3) cannot be an equilibrium.

On the other hand, if \( b > \frac{1}{2} \), country A will set \( t < \min \left\{ \beta, \frac{2b-1}{b}, t^L \right\} \) and specialize in good 1 while country B diversifies (case 4). Now the tax rate has to be sufficiently low, so that country A produces only the polluting good. It then obtains the welfare level \( u_{case\ 4}^A (b) \), which exceeds the welfare it could get from the alternative patterns of specialization (cases 5 and 6). Since \( t < t^L \) (respectively, \( t < \frac{2b-1}{b} \) ), case 5 (respectively, case 6) cannot be an equilibrium.
References


**Figure 1:** Global Production Possibility Frontier (PPF) under alternative patterns of specialization

![Graph showing the Global Production Possibility Frontier (PPF) for Country A and Country B, with different patterns of specialization.](image)

**Table 1:** Country A’s optimal pollution tax rate under autarky, $\tilde{t}$, for selected values of $b$ and $\beta$

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