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Modeling the Optimization Problem of a Public University*

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Abstract: In this paper, we examine the factors that influence the reputation or prestige of a public university. We develop a model of university behavior that indicates how the decisions made by university officials would be chosen in order to maximize their respective reputations. In doing so, we assume that reputation is enhanced by the quality of teaching and research produced as well as the service provided to the community in terms of the provision of publicly funded education services. We argue that the relative weights placed on these intermediate outputs may vary by university type as well as the means of producing them. Given the optimization problem of a university, a number of interesting conditions governing the policy variables chosen by the officials of public universities are obtained.

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1. Introduction

In many countries, the state plays a significant role in providing post-secondary education to its citizens. For example, both Canada and the United States feature an extensive network of what might be called public or non-profit universities. In these and other countries, government officials and interested observers often wonder what are the precise objectives of these public universities. In particular, given the myriad types of public funded universities, there is often a significant difference in behavior and operation of these institutions. While there has been significant research on the behavior and performance of public firms, there has been less attention paid to the behavior and performance of public universities.

In looking at the stated objectives of universities they are usually described in “Mission Statements” which invariably refer to the contribution to knowledge, both in the flow of ideas transmitted to students and in expanding the stock through research. A common view is that a university’s reputation or prestige is its most important goal.\(^1\) For instance, the vision expressed by Queen’s University was to be the “quality leader in Canadian higher education”.\(^2\) That managers in higher education appear motivated by “the pursuit of excellence” is a view also shared by academics who have studied the economics of higher education.\(^3\) Given these views, it would appear the issue of how these general statements of purpose translate into more concrete objectives and operating strategies would be a significant academic and policy question.

Given this apparent goal, the interesting issue is: how do universities enhance their reputation or prestige? It has been observed that a university can be viewed as a multi-product firm that produces a variety of outputs, which include teaching, research and “community service”.\(^4\) While these common outputs are shared by almost all universities, a significant diversity among institutions in terms of these outputs prevails. This difference includes the relative emphasis placed upon teaching and research, the

\(^1\) This was the finding of a 1982 survey of the views of the Presidents and Board Chairs of 50 Canadian universities. See University of Alberta (1982), p. 144.
\(^2\) Queen’s University (1999).
\(^3\) See Winston (1999) p.16 for a further discussion of this theory. See also Brewer, Gates and Goldman (2002) and Clotfelter (1996) for further discussions of the objectives of university and college officials. For a thorough examination of a number issues related to higher education in Canada, see Beach et. al. (2005).
\(^4\) For further discussion of this issue, see Cohn et al. (1989) and Dickson (1997).
size of the university, the diversity in fields of instruction ranging from liberal arts to professional and vocational training, and the extent of undergraduate and graduate training.

This difference in mission was highlighted by Brewer et al. (2002) in their examination of the U.S. higher education system, in which they focused on the strategic choices and competitive behavior of universities. Based on extensive interviews of officials from 26 diverse institutions over a two year period, Brewer et al. (2002) classify US universities into prestigious institutions (PI), prestige seeking (PS) and reputation-based (RB). Briefly, Prestigious Universities, or PI type institutions are confident that students will be drawn to them based on their perceived excellence. These universities are not actively engaged in increasing the number of potential students but are more interested in increasing their market share of strong students and faculty. In contrast, Prestige Seeking Universities (PS) universities are interested in enhancing their prestige and therefore choose to devote a large fraction of their budget to enhancing their relative standing. PS universities are not as constrained by past practice as PI universities and therefore are willing to experiment with their educational programs in the hopes of finding a successful formula. Reputation Based, or (RB) type universities, have neither a high level of prestige nor are they particularly interested in making the necessary investments to acquire it. According to Brewer et al. (2002), these institutions are more concerned with meeting the demands of students at a reasonable price. These schools tend to describe their goals in relation to the needs of the external constituents.

Similar to the classification used by Brewer et al. (2002), the most commonly adopted approach used in Canada is to classify universities as Medical/Doctoral, Comprehensive universities or Primarily Undergraduate Institutions. The distinction here is between universities with medical and doctoral programs, comprehensive universities who do not have medical schools and but have a limited number of doctoral programs, and primarily undergraduate institutions which have neither medical schools or doctoral programs but may have a selected number of masters programs. It can be argued that this classification corresponds roughly to the classification of Prestigious, Prestige Seeking and Reputation-Based Universities used by Brewer et al. (2002).

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5 This is the classification used by Maclean’s Magazine in its annual rankings of Canadian Universities.
In this paper, we develop a model of university behavior for a public university based on the different types of universities described above. We specify a particular objective function which allows for a range of preferences for university officials. These objectives are pursued through a number of university decisions regarding teaching, research and university size. For a public university these decisions must be made within a budget constraint that is mandated by governments. Based on an optimization problem we set out for university officials, we are able to determine a number of decision rules that might govern their choices which provides some insight into their behavior. The results here can be interpreted as a set of normative rules for university officials, but the latter part of the paper includes a discussion of how the general model might be formulated for empirical work.

This paper is organized as follows. In Section 2, we outline the public university’s objective function. We specify how teaching, research and university size relate to a university’s objectives. In Section 3, we outline the public university’s budget constraint in detail, while in Section 4, we outline the university’s optimization problem and derive a number of conditions governing its policy choices. Section 5 outlines a number of policy rules for university officials while Section 6 concludes the paper.

2. Modelling the Public University's Objective Function

In order to specify the objective function of an institution, it is important to specify the decision maker. For a public university, there are several possibilities, including the President, Senior Administration, the Board of Regents, and the Senate. For Canadian Universities, it is often the President (along with Senior Administration) who sets the broad policy agenda for a university, with financial issues being the purview of the Board of Regents, and academic issues vetted by the Senate. Given that the President and Senior Administration usually set the agenda for future academic and financial developments, one can make the argument that the President, along with Senior Administration, decide the overall academic and financial direction of the University.

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6 The following economic model is based largely on the Canadian university system, but it is clear that there are strong parallels with the State university system in the U.S. For a good overview of the financial structure and accounting of Canadian universities, see AUCC (2008).

7 This is not to say the President is not influenced by members of the Board of Regents and Senate, but in general, both academic and financial issues require support of Senior Administration, which is led by the President.
Accordingly, we assume the objective of the public university, as reflected by the direction set by the President and Senior Administration, is to maximize the institution's reputation, or relative ranking (P). The reputation of the public university (P) is assumed to be based on the research output (R), teaching output (T) and the size of the institution (S). Both the research output and teaching output are adjusted for quality. The institution’s size may be interpreted as a scale factor associated with research and teaching, or following Garvin (1980), the contribution to the community in providing education services to a large number of students. Mathematically, the reputation or prestige function is given as

\[ P = P(R, T, S) \]  

with the derivatives \( P_R > 0, \ P_{RR} \leq 0, \ P_T > 0, \ P_{TT} \leq 0 \) and \( P_S > 0, \ P_{SS} \leq 0 \). While each of these intermediate outputs, R, T and S are assumed to increase the prestige of the University, the relative importance of these outputs, in our model, is determined by the President and Senior Administration. As can be seen, the above formulation allows for research output (R) and teaching output (T), to interact with the size of the institution (S), to produce a university’s reputation. This allows for a trade-off between research, teaching and the size of the institution.

A specific functional form that has the above properties is \( P = R^{\omega_r} T^{\omega_t} S^{\omega_s} \), where \( \omega_r, \omega_t \text{ and } \omega_s \) are the respective weights placed on the intermediate outputs of research, teaching, and size of the institution. We assume that \( 0 \leq \omega_r, \omega_t \leq 1; \ 0 \leq \omega_s \leq 1 \) and \( 0 \leq \omega_t, \omega_s \leq 1 \). Given that a university’s reputation or prestige is generally viewed as an ordinal ranking, a specific function version of (1) is the following.

\[ \ln P = \omega_r \ln R + \omega_t \ln T + \omega_s \ln S \]  

\[ (1') \]

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8 For an empirical model based on a reputational model for universities see Cyrenne and Grant (2009).
10 Winston (1999:16) attributes the idea that universities have the objective of “prestige maximization” to James (1990). Clotfelter (1996) posits that managers in higher education are motivated by “the pursuit of excellence”.
11 This idea suggests that expanding enrolment is not without cost to research and teaching. Higher enrolments mean larger class sizes which implies that faculty may spend more time managing their respective courses. To the extent that higher enrolments are the result of lower admission standards, the quality of the classroom experience also suffers.
12 This allows for the possibility of a range of institutions regarding research or teaching quality in terms of size, depending on the resources available to pursue these goals. An alternative functional form might be linear, of the form, \( P = \omega_r R + \omega_t T + \omega_s S \). This form yields similar decision rules as a multiplicative form, but suggests the effect of the “inputs” on prestige can be separated.
13 These restrictions imply there are constant returns to reputation for these three outputs which we think is a reasonable assumption.
In determining the effect of the choice variables on the reputation function, one needs to decide how these university policy variables affect the various elements of (1) or (1’). In doing so, we identify two types of faculty members, faculty members of quality \(i\) hired to engage in research and teaching (\(L_i\)), and faculty hired for teaching purposes only (\(L_t\)). The former group can include faculty hired on tenure track contracts, tenured faculty, as well as recently recruited faculty.

It is also important to specify the relationship between universities. We assume that our public university makes its policy choices without taking account, in a strategic sense, the behavior of other universities. That is, we do not specify a non-cooperative game between universities.\(^{14}\) Finally, the public university in our model is required to maintain a balanced budget on its operations.\(^{15}\) The intermediate outputs \(R\), \(T\) and \(S\) are produced using a variety of inputs described in the following section.

### 2.1 Research Output

The research output of a university can be measured in a number of ways. Perhaps the most common definition is the number of quality adjusted research publications.\(^{16}\) We assume the research output of a university is a function of the number and quality of university faculty, as well as the resources available for research per faculty member (both internal and sponsored) as well as other expenditures spent in support of research. In order to capture the effect of faculty quality we define \(L_i\), as the number of research faculty of type \(i\). The range of research quality types might be specified as \([1,n]\) where \(n\) is the

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\(^{14}\) This would require that the policy variables for universities be jointly determined requiring some type of noncooperative equilibrium. A noncooperative game between universities would require that all policy variables for all universities be jointly determined. We feel that we can incorporate the effect of other universities on the policy choices of a university through the perceived supply responses to a university’s policy changes. For example, a public university operating in a local area, usually has some idea the effect of changes to its policy choices, for example, tuition levels and admission standards, has on its enrolment, given the presence of neighbouring institutions.

\(^{15}\) This might be also be viewed as a not for profit university. For a paper that also looks at the choices of a not for profit university, see Coates and Humphreys (2002) who examine the choices by a utility maximizing bureaucrat in the Niskanen (1975) tradition. We discuss the differences of our model with Coates and Humphreys (2002) at the end of this paper.

\(^{16}\) An alternative definition is the dollar amount of funded research, either sponsored or non-sponsored research. We prefer the research output definition, since it is our belief that research funding is based largely on the quantity and quality of peer reviewed research produced by research faculty. For an example of a research ranking methodology for Economics departments and journals see Kalaitzidakis, et. al (2003). For a much broader set of journals see ARC (2012) which is a publication of the Australian Research Council that ranks journals from a wide variety of disciplines in terms of their research quality.
highest ability research faculty. The wage rate is assumed to be increasing in the quality of the research faculty member.

In order to simplify the exposition, we assume the research output of a university is a function of the number and quality of research faculty \((L_i)\) as well as the average quality of graduate students, \(Q_g\). We assume that faculty quality is homogenous at a particular university. Regarding the average quality of graduate students \((Q_g)\) it can be measured by the average undergraduate GPA adjusted for the quality of the student’s undergraduate university.

For our purposes, it is also important to separate the overall research output at the university from the scale or “size” of the university. In order to do so we define \(FS_i = \frac{L_i}{FTET}\), which is the faculty-student ratio, or the ratio of research faculty to total enrolment. The idea here is that the overall research output of a university should be adjusted for the size of the institution. For example, we argue that a given number of research publications is more impressive if produced by a smaller institution. Since we would expect larger institutions to do more research, we argue a good measure of research output should take into account the scale or size of the university. This also allows us to avoid double counting since it is important to separate the size of the university from research and teaching output in order to provide a separate role for the “size” of the university. This is consistent with the argument often made that university officials face trade-offs between quality (in terms of teaching and research) and quantity (the size of the institution.)

Thus, the research output of the university, in per capita terms, is then,

\[
R = R(FS_i, Q_g) \tag{2}
\]

It is helpful to use the following specific function for (2) which is \(R = (L_i/FTET)^{\alpha_i} Q_g\). Whether there are increasing or (diminishing) returns to research faculty (of a given quality) depends on whether \(\alpha_i > 1\) or \((\alpha_i < 1)\). The average quality of graduate students is specified as \(Q_g = Q_g(as_g, s_g, P_{t-1})\) where \(as_g\) is the admission standard for graduate students, \(s_g\) is the level of graduate scholarships and \(P_{t-1}\) is the reputation (or prestige) of the University in the previous period.\(^\text{17}\) A specific function for the average quality of graduate students properties is \(Q_g = as_g^{aa} s_g^{ag} P_{t-1}^{ap} \).\(^\text{18}\)

\(^\text{17}\) It is also possible to include the number of research quality of faculty type i as a factor that influences the average
Substituting for the determinants of the average quality of graduate students in the expression for \( R \) yields, \( R = \left( \frac{L_i}{FTET} \right)^{\alpha_i} \text{as}_g \text{ag} P_{t-1}^{\alpha_p} \), which in natural ln form, yields the research output function

\[
\ln (R) = \alpha_i \ln L_i + \alpha_a \ln \text{as}_g + \alpha_s \ln \text{ag}_g - \alpha_i \ln FTET + \alpha_p \ln P_{t-1}
\]

with the \( 0 < \alpha_i < 1, 0 < \alpha_a < 1, 0 < \alpha_s < 1, 0 < \alpha_p < 1. \) The signs of the parameters suggest that the research output of the public university is increasing in the number of research faculty, admission standards, undergraduate scholarships, and the previous reputation of the university. As can be seen, total enrolment has an adverse effect on a university’s research output, given the term \(-\alpha_i \ln FTET.\)

### 2.2 Teaching Output

In general, determining undergraduate teaching output (adjusted for quality) is more difficult than estimating research output. It is generally thought that the quality of undergraduate teaching, or value added depends on the quality and quantity of faculty members, the average quality of undergraduate students \( (Q_u) \), the number of students (which affects the faculty student ratios), and the support provided by the university for teaching. We can define two faculty student ratios, \( FS_i = \frac{L_i}{FTET} \) (defined previously) and \( FS_t = \frac{L_t}{FTET} \) which is the ratio of teaching faculty to total enrolments. Given the above, we assume that teaching output or value added (in per capita terms) of a public university is given as

quality of graduate students, but we prefer to use the lagged value of the university’s reputation \( (P_{t-1}) \) instead. Since the two variables are highly correlated, and it simplifies the algebra, we think this is a preferable modeling approach.

It is possible that the \( Q_g \) of graduate students could be a function of the quality and quantity of research faculty, either contemporaneous or lagged; however, we feel the lagged reputation of the university is a significant and crucial determinant of the quality of graduate students attracted to a university.

The prior reputation or prestige of the university can affect research quality in a number of ways, by attracting superior faculty, superior graduate students and more external research funds. In the model, we choose to focus on the role of the prior reputation of the university \( P_{t-1} \), in attracting stronger undergraduate and graduate students, to a university. It is clear this variable plays a key role in an empirical model of university reputation.

The issue of defining teaching quality has attracted much discussion. For alternative views, see Green (1994), Darling-Hammond (1997), Harvey and Knight (1996), and Barnett (1992). This literature also relates to the extensive literature the student evaluation of teaching (SET), see for example Broder and Dorfman (1994).

While it is possible that teaching quality might include the quality of graduate instruction, in general, we feel that important trade-off between universities is between research quality and undergraduate teaching. In general, undergraduates pursuing graduate studies are often unaware of the quality of graduate instruction, and are more concerned with the research quality of faculty.

Working with faculty student ratios, enables us to separate the teaching quality from the scale or size of the university.
\[ T = T(FS_t, FS_t, Q_u) \] (3)

Regarding teaching, we specify a teaching production function, as \( T = (L_t/FTET)^{\gamma_t} (L_t/FTET)^{\gamma_r} \) with \( (\gamma_t, \gamma_r, < 1) \) where the first two terms are the respective faculty/FTET ratios for teaching and research faculty, and \( Q_u \) is the average quality of undergraduate students. As can be seen this function exhibits diminishing returns to teaching quality as the respective faculty/student ratio increases for both teaching and research faculty members.

Much like the average quality of graduate students, the average quality of undergraduate students \( (Q_u) \) could be measured by the average incoming high school grade (adjusted for the quality of the student’s high school), or the average score on a standardized test like the SAT. The average quality of undergraduate students \( (Q_u) \) is assumed to be positively related to the admission standards for undergraduates \( (a_{su}) \), the level of undergraduate scholarships \( (s_u) \) and the reputation (or prestige) of the university in the previous period \( (P_{t-1}) \) or \( Q_u = a_{su} s_u P_{t-1} \). A specific function with these properties is \( Q_u = a_{su}^{\gamma_a} s_u^{\gamma_s} P_{t-1}^{\gamma_p} \). Substituting the determinants of average quality of undergraduate students into the expression for teaching output or value added yields, \( T = (L_t/FTET)^{\gamma_t} (L_t/FTET)^{\gamma_r} a_{su}^{\gamma_a} s_u^{\gamma_s} P_{t-1}^{\gamma_p} \) which in natural log form yields the teaching output or value added function

\[
\ln T = \gamma_t \ln L_t + \gamma_r \ln L_r + \gamma_a \ln a_{su} + \gamma_s \ln s_u - (\gamma_t + \gamma_r) \ln FTET + \gamma_p \ln P_{t-1} \quad (3')
\]

with \( 0 < \gamma_t < 1, \ 0 < \gamma_r < 1, \ 0 < \gamma_a < 1, \ 0 < \gamma_s < 1, \ 0 < (\gamma_t + \gamma_r) < 1 \) and \( 0 < \gamma_p < 1 \).

Similar to research output, higher total enrolment reduces the quality of teaching output at the university at the rate \( -(\gamma_t + \gamma_r) \ln FTET \) which captures the adverse effect of higher enrolments on the teaching effectiveness of faculty.

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23 Much like the case of the average quality of graduate students, we could include the quality of faculty here as a factor influencing the average quality of undergraduate students; however, the algebra is much simplified if the lagged reputation is included instead.
2.3 Size of Institution

The size of the institution, $S$, could be interpreted as either the quantity of teaching and research output or the quantity of education services provided to the community. Our approach is to try to separate the quality dimensions of research and teaching, from the overall scale or size of the institution. This suggests that university officials might be willing to make trade-offs between research quality, teaching quality and the size of the university. The approach taken here is to use the total enrolment or $FTET$ as a measure of institutional size.\textsuperscript{24} Specifically, we define $S$, the size of the university as the total enrolment in terms of $FTET$ (Full Time Equivalent Students) which is the sum of undergraduate enrolment ($FTE$) and graduate enrolment ($GRFTE$).\textsuperscript{25}

$$S = FTET = FTE + GRFTE$$

(4)

Enrolment at a university depends on a combination of demand side variables, for example tuition and scholarships, and supply side variables, which includes non-price rationing (admission standards) as well as the university’s capacity to admit students, which is related to the number of faculty as well as a number of exogenous factors affecting enrolment, captured by the vector $z$.\textsuperscript{26} In modelling enrolments, we specifying enrolment functions for both undergraduate and graduate students, which are described as follows.

(i) Undergraduate Enrolment and Graduate Enrolment

Mathematically, the undergraduate enrolment function is given as\textsuperscript{27}

$$FTE = FTE(s, t, n, FTE_{t-1}, z)$$

(4')

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\textsuperscript{24} Essentially students are both inputs (stronger students enhance teaching) and outputs (in terms of increasing university size). For a model that takes this approach see Rothschild and White (1995).

\textsuperscript{25} It is possible that a university’s size might involve a different weighting of undergraduate versus graduate enrolment.

\textsuperscript{26} We assume each university can estimate of the effect of changes to these variables on enrolment.

\textsuperscript{27} For a discussion of issues related to estimating enrollment elasticities, see Fortin (2005). Our enrolment function differs from other work in the area in that we include supply factors (faculty numbers, admission standards) as well as demand factors (tuition fees, and scholarships). That is, we are working with reduced form enrolment functions.
where $s_u$ is the undergraduate scholarship amount (per FTE), $t_u$ is the undergraduate tuition rate (per FTE), $as_u$ is the undergraduate admission standard (in terms of the minimum high school grade criteria), $nu$ is the number of undergraduate course offerings, $FTE_{t-1}$ is the undergraduate enrolment in the previous period and $z_u$ is a vector of exogenous factors effecting enrolment. We assume that the number of undergraduate course offerings $n_u$ is a function of the number of faculty both research $L_i$ and teaching $L_t$, that is $n_u = g(L_i, L_t)$. Regarding the role of course offerings on enrolment ($\partial FTE/\partial n_u$), we include this term since in general the greater the number of faculty, the greater are the respective course offerings that are possible, which can lead to higher enrolments.

Regarding, the undergraduate enrolment function, we specify the following relationship $FTE = s_u t_u^{-\beta_s} as_u^{-\beta_a} FTE_{t-1}^{-\beta_o}$. Regarding the number of undergraduate courses offered ($n_u$), we use the following specification, $n_u = L_i u_t L_t u_r$, with $u_t \leq 1$ and $u_r \leq 1$. Substituting into the above enrolment function, yields $FTE = s_u t_u^{-\beta_s} as_u^{-\beta_a} FTE_{t-1}^{-\beta_o}$ or $FTE = s_u t_u^{-\beta_s} as_u^{-\beta_a} L_i u_t L_t u_r FTE_{t-1}^{-\beta_o}$ with the signs of the parameters reflecting the relationships assumed in the model. It is important to note that the parameters $\beta_{ut}$ and $\beta_{ur}$, contain both the effect of increased research and (teaching) faculty on course offerings, the parameters $(u_t)$ and $(u_r)$, and the effect of increased course offerings on enrolment ($\beta$).

Given the above, the undergraduate enrolment function (4') can be transformed as

$$ln FTE = \beta_t ln t_u - \beta_s ln s_u - \beta_a ln as_u + \beta_{ut} ln L_i + \beta_{ur} ln L_t + \beta_o ln FTE_{t-1}$$

In much the same way as for undergraduate enrolment, we also consider a graduate student enrolment function, which is for simplicity,

$$GRFTE = GRFTE (s_g, t_g, as_g, n_g, GRFTE_{t-1}, z_g)$$

indicates that total graduate student enrolment is a function of the level of graduate scholarship $s_g$ (per FTEGR), graduate tuition $t_g$ (per FTEGR), $as_g$ the graduate admission standards, and the number of

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28 Regarding scholarships, this might be viewed as an entrance scholarship, or a scholarship for continuing students.
29 It might be thought that higher admission standards might lead to higher enrollment; however, we assume that enrollment is decreasing in admission standards, holding the reputation of a university constant.
30 Although, we have been unable to find any empirical work related to this issue, evidence of this effect is often seen when a university or department budget is reduced. If sessional faculty are not re-hired, this generally results in a reduction in course offerings, with lower enrolment in a department.
31 In general, research faculty have lower teaching loads than teaching faculty which limits the number of new undergraduate courses that result from increased research faculty.
graduate courses $n_g$ (which is a function of the number of research and teaching faculty $n_g = h(L_t, L_i)$, previous graduate enrolment ($GRFTE_{t-1}$) and other exogenous factors effecting graduate enrolment ($z_g$).

Similar to the role of course offerings on undergraduate enrolment, we have also included the effect of an increased number of graduate programs on graduate student enrolment (which is possible with an increased number of research and teaching faculty).\(^{32}\)

In much the same way, the graduate enrolment function (5') specifies a relationship between the number of graduate programs offered and the number of faculty hired. For simplicity we specify the function $n_g = L_i^{gr} L_t^{gt}$, with $gr, gt \leq 1$. Given this relationship, we can specify the graduate enrolment function $GRFTE = s_g^{\delta_g} t_g^{-\delta_t} a_{sg}^{\delta_a} n_g^{\delta_n} GRFTE_{t-1}^{\delta_o}$ or $GRFTE = s_g^{\delta_g} t_g^{-\delta_t} a_{sg}^{\delta_a} L_i^{\delta_l} L_t^{\delta_l} GRFTE_{t-1}^{\delta_o}$, which can be transformed as\(^{33}\)

$$\ln GRFTE = \delta_g \ln s_g - \delta_t \ln t_g - \delta_a \ln a_{sg} + \delta_{gr} \ln L_i + \delta_{gt} \ln L_t + \delta_o \ln GRFTE_{t-1}$$  \hspace{1cm} (5'')

Regarding the effect of an increase in research faculty on graduate enrolment, the parameter ($\delta_{gr}$), contains both the effect of increased research faculty on graduate program offerings - the parameter ($gr$) - and the effect of increased graduate programs on graduate course enrolments ($\delta$).

Using $\ln S = \ln FTET + \ln FTE + \ln GRFTE$. \hspace{1cm} (4''') and (5'') yields\(^{34}\)

$$\ln S = \beta_s \ln s_u + \beta_t \ln t_u - \beta_a \ln a_{su} + \beta_{ur} \ln L_i + \beta_{ut} \ln L_t + \beta_o \ln FTE_{t-1} +$$

$$+ \delta_g \ln s_g - \delta_t \ln t_g - \delta_a \ln a_{sg} + \delta_{gr} \ln L_i + \delta_{gt} \ln L_t + \delta_o \ln GRFTE_{t-1}$$

which if $\beta_o = \delta_o = \gamma$, we can let $\ln S_{t-1} = \ln FTE_{t-1} + \ln GRFTE_{t-1}$ which yields, collecting terms

$$\ln S = \beta_s \ln s_u - \beta_t \ln t_u - \beta_a \ln a_{su} + \delta_g \ln s_g - \delta_t \ln t_g - \delta_a \ln a_{sg} +$$

$$(\beta_{ur} + \delta_{gr}) \ln L_i + (\beta_{ut} + \delta_{gt}) \ln L_t + \gamma \ln S_{t-1}$$  \hspace{1cm} (6)

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\(^{32}\) An alternative modeling approach would be to have the public university choose the number of courses and programs as; however, in many cases, these are determined by the availability of teaching and research faculty. In some cases, the introduction of new programs by a public university must be approved by a provincial or state authority.

\(^{33}\) We might also include a vector of other factors $z_g$, for example the location, or age of the University. We assume that entry quality declines with the number of students admitted (Garvin, 1980) and spending on scholarships can interpreted as the extent of price or tuition discrimination (Danziger, 1990).

\(^{34}\) Both terms for lagged enrolment are predetermined variables for the university and so do not affect the optimization decision of universities. They do play an important role; however, in empirical models estimating enrolments for universities.
(iv) **Indirect Reputation Function**

If we substituting (2’) and (3’) into (1’) yields what might be called an Indirect Reputation Function, listed below.

\[
\ln P = \omega_r \left( \alpha_i \ln L_i + \alpha_a \ln a_s + \alpha_s \ln s_g - \alpha_r \ln S + \alpha_p \ln P_{t-1} \right) \\
+ \omega_t \left( \gamma_i \ln L_t + \gamma_a \ln L_i + \gamma_a \ln a_s + \gamma_s \ln s_u - \left( \gamma_r + \gamma_p \right) \ln S + \gamma_p \ln P_{t-1} \right) + \omega_s \ln S
\]

Grouping terms in \( \ln S \) yields

\[
\ln P = \omega_r \left( \alpha_i \ln L_i + \alpha_a \ln a_s + \alpha_s \ln s_g + \alpha_p \ln P_{t-1} \right) \\
+ \omega_t \left( \gamma_i \ln L_t + \gamma_a \ln L_i + \gamma_a \ln a_s + \gamma_s \ln s_u + \gamma_p \ln P_{t-1} \right) \\
+ \left( \omega_s - \omega_r \alpha_r - \omega_t \left( \gamma_r + \gamma_p \right) \right) \ln S
\]  

(7)

As can be seen the policy parameters, \( L_i, a_s, s_g, L_t, a_s, s_u \) have in general, a direct effect on research and/or teaching as well as an indirect effect on \( P \), through their effect on the size of the institution (S) as outlined in (6). The term in brackets (\( \omega_s - \omega_r \alpha_r - \omega_t (\gamma_r + \gamma_p) \)) includes the direct effect of size on prestige \( \omega_s \) (which is determined by the weight assigned to size), but also the adverse effects of larger enrolments on research (\( -\omega_r \alpha_r \)) and teaching (\( -\omega_t (\gamma_r + \gamma_p) \)). These latter terms are needed in order to capture the overall effect of size and on both teaching and research, and in essence, they allow the first two terms of (7) to be scale adjusted measures of research and teaching quality. The additional policy variables \( t_g, t_u \) have a direct effect on enrolment.

Equation (7) provides a specific functional form for the university’s reputation that corresponds to the normative model derived previously. Combined with the university’s budget constraint, and with the appropriate data, it can form the basis for an empirical model of university decision making. A similar estimation problem was undertaken by Cyrenne and Grant (2009), which was designed to uncover the factors which influenced the ranking of universities in Canada.
3.0 The University’s Budget Constraint

Our public university is assumed to maximize its prestige or reputation subject to a break even operating constraint. Unlike the United States, Canada’s university system is essentially wholly public.\(^{35}\)

In a sense, the Canadian university system might be viewed as comparable to the system of State universities as exists in the U.S., without the extensive private college and private university sector which plays a large role in the U.S. higher education system.

A key issue in modelling a university’s optimization problem is the role of tuition fees. In some jurisdictions in the United States, individual public institutions are able to set their own tuition levels. However, in its Triennial Survey (2005-2006), the State Higher Education Officers (SHEEO) found that only in 14 states were public institutions allowed to set their own tuition.\(^{36}\) Moreover, in only 5 of those 14 states were individual public institutions allowed to set tuition rates with no external restrictions.\(^{37}\) In the remaining states, the primary authority for establishing tuition remained with the legislature (5 states), a state-wide agency (13 states), a coordinating/governing board for individual systems (23 states) and local district governing boards 93 (3 states).\(^{38}\)

In Canada, higher education is a provincial responsibility and provinces regulate tuition levels, in one form or another, in all Provinces. As outlined by the Association of Universities and Colleges of Canada (AUCC (2008:28)), provincial government support for universities in Canada is often tied to freezes or tight controls on tuition fees. In effect, provincial governments view increased grants to universities as a way of offsetting some of the revenue losses that have resulted from new tuition regulations or agreements with universities regarding tuition policies. Recent restrictions on allowable tuition fees include tuition freezes in Quebec, Newfoundland and Manitoba, as well as in Saskatchewan and Alberta in 2004, and lately, Prince Edward Island, Nova Scotia and New Brunswick.\(^{39}\) In addition,

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\(^{35}\) While one might debate the meaning of public versus private, for our purposes “public” means receiving a substantial fraction of the university’s budget from government grants, as well as provincial government, or state representation on the Board of Governors of the university.

\(^{36}\) See SHEEO (2006: 9) for a list of states and tuition setting authority.


\(^{38}\) See SHEEO (2006:9), Table 2.

\(^{39}\) In Newfoundland and Manitoba, the tuition fees also included rollbacks in student fees. See AUCC (2008:28).
the governments in British Columbia and Ontario have imposed tuition freezes at various times in the last
decade, while at other times, the tuition fee increases are regulated.

Despite the examples given above, in order to provide a more general model, we assume the
public university is able to set its own tuition level. Our model is easily amended; however, for
jurisdictions in which the tuition level is exogenously set. In this case, the set of endogenous variables in
the optimization problem is reduced, with the result that all endogenous variables in equilibrium would be
functions of the exogenously set tuition levels.\textsuperscript{40} With tuition exogenously set, it is clear that university
officials are more dependent on enrollment and external fundraising to balance their budgets. This leads,
however, to an adverse effect on teaching and research quality, since larger enrollments decrease faculty
student ratios.

Given the above, it is useful to examine the budget of a representative public university in
Canada.\textsuperscript{41} There are two types of budgetary measures that might be considered, the larger Total
University Income and Total University Expenditure measure, or the smaller General Operating Income
and General Operating Expenditure measure.\textsuperscript{42} It is our belief that, at least for public universities, that
the key budgetary measure for university decision making relate to the Operating Budget.

Regarding General Operating Income, there are 4 main revenue sources for Canadian universities,
which include, government grants, tuition revenue (undergraduate) and (graduate), charitable
fundraising, and sponsored research. In our model, we also include one additional revenue category,
which are monies for scholarships (which shows up in the Special Purpose and Trust category from the
General Income Accounts). This is money transferred from endowment funds to the operating side,
which can be included in income category I. We assume that government grants, tuition revenue and
charitable fundraising, are all related to enrolments. Regarding General Operating Expenditure, there are
9 principal categories, which are expenditures on Instruction (which includes faculty salaries), Student

\textsuperscript{40} In our model, the set of first order conditions would be reduced by two, conditions governing the University’s
choice of undergraduate \((t_u)\) and graduate tuition \((t_g)\).

\textsuperscript{41} The principle source for financial data on Canadian Universities, comes from CAUBO, which is the Canadian
Association of University Business Officers. Their publication provides a rich set of information on Canadian
universities. Balanced budgets are the norm for publicly funded universities in Canada. See also AUCC (2008)
which outlines the financial structure and reporting of Canadian Universities.

\textsuperscript{42} See Appendix I for a detailed discussion of the difference between these amounts. In order to abstract from
ancillary enterprises, we focus on the operating budget.
Services, the Physical Plant, Administration, Computing, Library, Non-credit instruction, Sponsored Research, and Scholarships.

Given the above, we can express the public university’s budget constraint as follows.

\[(g_u + d_u + t_u)FTE + (g_g + d_g + t_g)GRFTE + s_iL_i + I_o = w_iL_i + w_iL_i + s_uFTE + s_gGRFTE + \sum EXP_i \]

(8)

The left hand side of (8) includes the university revenues from undergraduate enrolment, \((g_u + d_u + t_u)FTE\), university revenues from graduate enrolment, \((g_g + d_g + t_g)GRFTE\), revenues from sponsored research (as a function of the number and quality of research faculty) \(s_iL_i\) and all other operating revenues, \(I_o\), which includes all other operating revenues (which includes transfers from endowment funds – called Special Purpose and Trust).  

Focusing on the first term on the left hand side of (8), total university revenues come from undergraduate tuition \((t_u)\) (based on enrolments), and government grants per FTE \((g_u)\), the latter which is an example of a specific funding formula for a public university. A funding agreement for public universities based on enrolment is used in a number of jurisdictions. Also included is a charitable donation per FTE \((d_u)\). While it is unlikely that potential donors tie their donations to the number of FTEs admitted in a university, this might be thought of as an “implicit” donation rate. The assumption here is that total donations are a function of enrolments, with a linear relationship assumed for convenience. 

Regarding the second term on the left hand side of (8), we also assume there is government support for graduate students \((g_g)\) based on GRFTE. In addition, universities are assumed to collect graduate tuition \((t_g)\) as well as donations \((d_g)\) which may be thought of as monies for non-sponsored research.

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43 This ‘catch all” category can include all other sources of revenue that can be used for operating purposes, including ancillary enterprises and athletics programs of the Public University.

44 For a discussion of varying funding formulas used by governments to fund higher education, with an emphasis on North America, see Lang (2005), McKeown (1996), McKeown-Mack (1999) and Marks and Caruthers (1999) and Brons (1990). For details of the funding of universities in the Netherlands, see Koelman (1998).

45 For a discussion of the role of charitable donations for nonprofit firms like public universities, see Winston (1999:16).

46 It is important to note that we do not include athletics as a revenue or a expenditure category explicitly. To the extent athletics is a net expense it could be included as one of the expenditure categories which enhances reputation.
The right hand side of (8), or the Operating expenditures for a public university, is straightforward and includes total salaries for teaching faculty \((w_tL_t)\) and research faculty \((w_iL_i)\) which are included in the Instruction expenditure category above with all other operating expenditures (except expenditures on Scholarships), expenditures on undergraduate \((s_uFTE)\) and graduate scholarships \((s_gGRFTE)\) and \(EXP\), which includes all other operating expenditures not included in the above.\(^{47}\)

It is possible to rewrite (8) by collecting terms which yields (8’), the University’s budget constraint as,

\[
(g_u+d_u+t_u\cdot s_u)FTE+(g_g+d_g+t_g\cdot s_g)GRFTE + I_o = w_tL_t + (w_i - s_i)L_i + \sum_{i} EXP_i
\]  

(8’)

In this formulation, the salaries of research faculty are net of the average amount of sponsored research per research faculty, \(s_i\), which can be thought of as revenues from sponsored research, which yields the total cost of research faculty as \((w_i - s_i)L_i\). Similarly, we can move the amount the expenditures on scholarships to the left hand side, reducing the net revenue form undergraduate and graduate enrolments. This makes it clear that scholarships are a “cost” to the University, in that it reduces the total tuition revenues.\(^{48}\) The term \((s_u)\), scholarships per \(FTE\) is actually an expenditure rate, which is easier to include on the income side, as reducing the revenues per \(FTE\) from the other categories.

4.0 The Public University’s Optimization Problem

Given the objective function, we assume that a public university chooses a number of policy variables in order to maximize its prestige (1) subject to the budget constraint (8’). Exogenous variables for the public university include the government grants per \(FTE\) \((g_u)\) and charitable donations per student \((d_u)\) as well as the wage rates for research faculty \((w_i)\) and teaching faculty \((w_t)\). Regarding graduate enrolments, exogenous variables include government grants \((g_g)\) and donations \((d_g)\). We also include a capacity level denoted as \(K\). The Lagrangean for this problem is then given as

Alternatively, a recent trend in Canada is for athletic programs like football to be run as a stand-alone enterprise with academic ties but no financial links to the university. To the extent athletics makes a net contribution to operating revenue it can be included in \(I_o\).

\(^{47}\) The expenditures on Scholarships are reflected on the left hand side of (8’) as \(s_uFTE\).

\(^{48}\) An alternative approach for a university would be to waive a fraction or all of a student’s tuition, (without awarding a dollar amount for a scholarship) as a method of awarding scholarships, essentially a tuition discrimination approach.
\[ \ell = P(R,T,S) + \lambda_1 [(g_u + d_u + t_u - s_u)FTE + (g_g + d_g + t_g - s_g)GRFTE + I_o - w_i L_i - (w_r, s_i)]L_i - \sum EXP_i + \lambda_2 [K - FTET] \]

The public university’s choice variables include its admission standards for both undergraduate \((a_s_u)\) and graduate students \((a_s_g)\) its undergraduate \((t_u)\) and graduate tuition levels \((t_g)\), the level of undergraduate scholarship \((s_u)\) per \(FTE\) and graduate scholarship \((s_g)\) per \(GRFTE\), as well as the number of faculty \((L)\) hired, or each type.\(^{49}\) We assume that there are two types of faculty members that a university can hire, faculty that are hired primarily for teaching (which includes stipendiary faculty and faculty with term appointments), denoted \((L_t)\) and faculty that are hired for their research potential, \((L_u)\). The salaries for the two academic markets are assumed to differ with the wage rate for research faculty \((w_r)\) assumed to be higher than the wage rate for teaching faculty \((w_t)\), and increasing in the quality of the faculty \(i\).\(^{50}\) We also assume the university chooses a number of operating expenditures, denoted \((EXP_i)\) in a number of areas designed to support teaching and research and general university operations, as outlined in the Appendix I.

The respective first order conditions are, assuming that the capacity constraint of the university is not binding, \(\lambda_2 = 0\),

\[
\begin{align*}
L_o: & \quad (\partial P/\partial R)(\partial R/\partial L_u) + (\partial P/\partial T)(\partial T/\partial L_u) + (\partial P/\partial S)(\partial S/\partial L_u) + \lambda_1 [(g_u + d_u + t_u - s_u)\partial FTE/\partial c_{n_u}, (\partial c_{n_u}/\partial L_u) - w_i] \leq 0 \text{ if } <0, L_o = 0 \\
t_o: & \quad (\partial P/\partial R)(\partial R/\partial t_u) + (\partial P/\partial T)(\partial T/\partial t_u) + (\partial P/\partial S)(\partial S/\partial t_u) + \lambda_1 [(g_u + d_u + t_u - s_u)\partial FTE/\partial c_{n_u} + \partial FTET] \leq 0 \text{ if } <0, t_o = 0 \\
as_o: & \quad (\partial P/\partial R)(\partial R/\partial a_s_u) + (\partial P/\partial T)(\partial T/\partial a_s_u) + (\partial P/\partial S)(\partial S/\partial a_s_u) + \lambda_1 (g_u + d_u + t_u - s_u)\partial FTE/\partial c_{a_s_u} \leq 0 \text{ if } <0, a_s_o = 0 \\
s_g: & \quad (\partial P/\partial R)(\partial R/\partial s_g) + (\partial P/\partial T)(\partial T/\partial s_g) + (\partial P/\partial S)(\partial S/\partial s_g) + \lambda_1 (g_g + d_g + t_g - s_g)\partial GRFTE/\partial c_{s_g} \leq 0 \text{ if } <0, s_g = 0 \\
EXP_i: & \quad \partial P/\partial EXP_i - \lambda_i \leq 0 \text{ if } <0, \text{ EXP}_i = 0 \quad i = 1, n \\
\lambda_1: & \quad (g_u + d_u + t_u - s_u)FTE + (g_g + d_g + t_g - s_g)GRFTE + I_o - w_i L_i - w_i L_u - \sum \text{EXP} \geq 0 \text{ if } >0, \lambda_1 = 0 \\
\lambda_2: & \quad K - FTET \geq 0 \text{ if } >0, \lambda_2 = 0
\end{align*}
\]

\(^{49}\) We model the decision making at the University level, for an analysis of decisions at the Department level, see Borooah (1994).

\(^{50}\) We assume the market for academics both research and teaching is competitive, with each university a price taker, with more productive researchers commanding a higher salary.
The first order conditions, assuming that the capacity constraint of the public university is not binding, yield a number of efficiency conditions for a public university. The first four conditions (9) to (12) relate to the undergraduate program while the set (13) to (16) correspond to the graduate program. Condition (9) indicates that teaching faculty \((L_u)\) are hired until the marginal effect on research, teaching and size \\
\\n\[
\left(\frac{\partial P}{\partial R}\right)\left(\frac{\partial R}{\partial L_u}\right) + \left(\frac{\partial P}{\partial T}\right)\left(\frac{\partial T}{\partial L_u}\right) + \left(\frac{\partial P}{\partial S}\right)\left(\frac{\partial S}{\partial L_u}\right)
\]
\\njust equals their respective marginal factor cost \((w_i)\) adjusted for the additional revenues brought in from additional undergraduate enrolments \\
\\n\[
(g_u + d_u + t_u - s_u)\left(\frac{\partial FTE}{\partial n_u}\right)\left(\frac{\partial n_u}{\partial L_u}\right).
\]
\\nCondition (13) indicates that research faculty \((L_i)\), are hired until the marginal effect on research, teaching and size \\
\\n\[
\left(\frac{\partial P}{\partial R}\right)\left(\frac{\partial R}{\partial L_i}\right) + \left(\frac{\partial P}{\partial T}\right)\left(\frac{\partial T}{\partial L_i}\right) + \left(\frac{\partial P}{\partial S}\right)\left(\frac{\partial S}{\partial L_i}\right)
\]
\\njust equals their respective marginal factor cost \((w_i)\) net of the average amount of sponsored research per research faculty \((s_i)\) adjusted for the additional revenues brought in from additional undergraduate \\
\\n\[
(g_u + d_u + t_u - s_u)\left(\frac{\partial FTE}{\partial n_u}\right)\left(\frac{\partial n_u}{\partial L_u}\right)
\]
\\nand graduate enrolments \((g_s + d_s + t_s - s_s)\left(\frac{\partial GFTE}{\partial n_s}\right)\left(\frac{\partial n_s}{\partial L_u}\right)\).
\\
Equation (10) indicates that undergraduate tuition \((t_u)\) should be raised until the marginal effect on reputation from increasing the level of tuition, \\
\\n\[
\left(\frac{\partial P}{\partial R}\right)\left(\frac{\partial R}{\partial t_u}\right) + \left(\frac{\partial P}{\partial T}\right)\left(\frac{\partial T}{\partial t_u}\right) + \left(\frac{\partial P}{\partial S}\right)\left(\frac{\partial S}{\partial t_u}\right)
\]
\\nplus the marginal benefit from the increased tuition revenue, \(\lambda_1\) \\
\\n\[
\left(\frac{\partial P}{\partial R}\right)\left(\frac{\partial R}{\partial \epsilon}\right) + \left(\frac{\partial P}{\partial T}\right)\left(\frac{\partial T}{\partial \epsilon}\right) + \left(\frac{\partial P}{\partial S}\right)\left(\frac{\partial S}{\partial \epsilon}\right)
\]
\\nmust eventually be greater than zero, otherwise, admission standards would be set to zero. \\
Equation (12) states that undergraduate scholarships should be increased until their respective marginal benefits in terms of enhanced reputation \\
\\n\[
\left(\frac{\partial P}{\partial R}\right)\left(\frac{\partial R}{\partial s_u}\right) + \left(\frac{\partial P}{\partial T}\right)\left(\frac{\partial T}{\partial s_u}\right) + \left(\frac{\partial P}{\partial S}\right)\left(\frac{\partial S}{\partial s_u}\right)
\]
\\just equals its marginal cost in terms of reduced revenues \\
\\n\[
\lambda\left[ FTE(g_u + d_u + t_u - s_u)\epsilon_{FTE, su} - s_u FTE \right] < 0,
\]
\\where \\
\\n\[
s_u FTE
\]
\\is the lost revenue to the university from awarding the scholarship and \\
\\n\[
FTE(g_u + d_u + t_u - s_u)\epsilon_{FTE, su}
\]
\\is the additional tuition revenue received from scholarships increasing enrolment, given the undergraduate

---

51 In general, the additional revenues brought in by hiring additional teaching and research faculty are less than their respective wage rate.

52 This implies that the benefit from higher admission standards on teaching must be greater than the loss from higher admission standards reducing the size of the university.
supply elasticity with respect to undergraduate scholarships $\epsilon_{FTE, u}$. The marginal benefit arises from higher scholarships increasing average student quality (and hence teaching quality) and the size of the university enhancing the university’s reputation.

From an examination of (15), we can see that a positive level of graduate student support ($s_g > 0$), requires the marginal benefit on reputation from increased graduate scholarships $[(\partial P/\partial R)(\partial R/\partial s_g) + (\partial P/\partial T)(\partial T/\partial s_g) + (\partial P/\partial S)(\partial S/\partial s_g)]$ plus the effect on university revenues $[(g_u + d_u + t_u - s_u)GRFTE/\partial s_g - GRFTE]$ must eventually be zero. Regarding the absence of graduate programs for some institutions, these universities may have chosen to focus on undergraduate teaching or they do not have a research environment that would merit a graduate program. Finally, equation (17) states that the marginal effect on reputation from each expenditure type must be equalized.

5.0 Policy Rules for University Officials

In order to get a clearer sense of the way in which the preferences of university officials interact with their budget constraint, it is helpful to use the specific formulations for the respective prestige, research, teaching and enrolment functions. Given the first order conditions (9) to (19), it is possible to outline a number of interesting policy rules for university officials. For example taking the ratio of (9) and (13), which define the optimal number of teaching and research faculty and rearranging yields

\[
\frac{\omega_r, \epsilon_{R,LT} + \omega_t, \epsilon_{T,LT} + \omega_s, \epsilon_{S,LT}}{\omega_r, \epsilon_{R,LI} + \omega_t, \epsilon_{T,LI} + \omega_s, \epsilon_{S,LI}} = \frac{[\omega, L_t - FTE (g_u + d_u + t_u - s_u) \epsilon_{FTE,LI}]}{[\omega, L_t - FTE (g_u + d_u + t_u - s_u) \epsilon_{FTE,LI} - GRFTE (g_s + d_s + t_s - s_s) \epsilon_{GRFTE,LI}]}
\]

(20)

where the parameters ($\omega_r$, $\omega_t$, and $\omega_s$) are the respective weights placed on research, teaching and university size and the parameters $\epsilon_{R,LT}$, $\epsilon_{T,LT}$ and $\epsilon_{S,LT}$ (for research, teaching and size) are the respective reputation elasticities with respect to teaching and research faculty.

Equation (20) states that teaching and research faculty should be hired until their respective weighted reputation elasticities with respect to teaching, research and size equals their respective total

53 Regarding the benefits of graduate students, it is clear that as more graduate students are admitted faculty members would be spending more time on supervision and less time on their own research, reducing the average quality of research. It is clear that at some point, the marginal benefit of increased numbers of graduate students must decrease otherwise the number of graduate students admitted would increase without limit.
wage bills (adjusted for sponsored research and the change in university revenues from increased faculty numbers). The change in revenue from hiring an additional faculty for teaching, through the effect of new course offerings attracting additional students, is \((FTE \,(g_u + d_u + t_u \cdot s_u) \, \varepsilon_{FTE,Li})\) while the corresponding amount for hiring a research faculty member, through the effect of new graduate programs attracting additional graduate students is \((GRFTE(g_g + d_g + t_g \cdot s_g) \, \varepsilon_{GRFTE, Li})\).

**Proposition 1.** The relative wage bill for teaching and research faculty, will be larger the larger the relative weights placed on teaching \((\omega_t)\) versus research \((\omega_r)\) and the larger the respective reputation elasticity of teaching faculty in relation to the respective reputation elasticity of research faculty.

Proof: Given (20), and assuming \(\varepsilon_{FTE,Li} = \varepsilon_{GRFTE, Li} = 0\) (no enrolment effects from increased teaching and research faculty for simplicity) yields
\[
\frac{\omega_r \varepsilon_{R, Li} + \omega_t \varepsilon_{T, Li} + \omega_s \varepsilon_{S, Li}}{\omega_r \varepsilon_{R, Li} + \omega_t \varepsilon_{T, Li} + \omega_s \varepsilon_{S, Li}} = \frac{w_t \cdot L_t}{(w_t \cdot s_t) \cdot L_t}.
\]
Given \(\varepsilon_{R, Li} < \varepsilon_{T, Li}\), and \(\varepsilon_{T, Li} > \varepsilon_{R, Li}\) by assumption, the greater, \(\omega_t, \varepsilon_{T, Li}\), but lower \(\omega_r, \varepsilon_{R, Li}\) or \(\varepsilon_{T, Li}\) the larger will be the respective wage bill for teaching versus research faculty.

Proposition 1 we feel is quite instructive. What is key to understanding the respective choice of teaching versus research faculty is not only preferences, but the ability of the respective faculty types to enhance teaching and research. Some schools may naturally place a higher weight on teaching, leading to lower expenditures on research faculty. However, other schools may place a relatively large weight on research \((\omega_r)\), but lack the environment to translate research faculty efforts into research output (a low \(\varepsilon_{R, Li}\) ) that is, research faculty may have a lower research productivity at that university. This can also lead to lower expenditures on research faculty.

**Proposition 2.** The ratio of admission standards to tuition level is a function of university preferences regarding research, teaching and size, as well as the respective elasticities of research, teaching and size with respect to increases in admission standards and tuition. This choice must also balance the respective effects of higher admission standards and tuition on university revenue.

Proof: Rewriting (8) and (7) and taking the ratio yields,
\[
\frac{\omega_r \varepsilon_{R, in} + \omega_t \varepsilon_{T, in} + \omega_s \varepsilon_{S, in}}{\omega_r \varepsilon_{R, in} + \omega_t \varepsilon_{T, in} + \omega_s \varepsilon_{S, in}} = \frac{((g_u + d_u + t_u \cdot s_u) \varepsilon_{FTE, inu})}{((g_g + d_g + t_g \cdot s_u) \varepsilon_{GRFTE, inu \cdot t_u})} \tag{21}
\]
Proposition 2 simplifies considerably in the case where (i) undergraduate admission standards or undergraduate tuition has no effect on research or teaching quality \((\varepsilon_{R,u}, \varepsilon_{T,u}, \varepsilon_{R,u} = 0)\) or (ii) university officials place no weight on research and teaching quality \((\omega_r, \omega_t = 0)\). In these cases, the left hand side of (21) reduces to the ratio of the two enrolment elasticities \((\varepsilon_{S,u}/\varepsilon_{S,u})\), which are the respective effects of lower admission standards and lower tuition on university enrolments (or size). In this case, admission standards and tuition are lowered until their respective elasticities with respect to university size equal their respective revenue effects.

The latter case, \((\omega_r, \omega_t = 0)\), might be viewed as a Niskanen (1975) effect for a public university. In Niskanen (1975), the objective of bureaucrats is to expand the size (or output) of the bureau until the total benefits from the bureau equal the total costs. Applied to our model, this result applies when university officials give little weight to research and teaching quality; suggesting they are solely interested in maximizing the size of the university.

**Proposition 3.** The amount of scholarship support for graduate versus undergraduate studies is dependent on the respective enrolment elasticities for graduate and undergraduate students, and their respective effects on university revenues.

**Proof:** Rewriting (13) and (9) from Appendix II and taking the ratio yields,

\[
\frac{(\omega, \varepsilon_{R,g} + \omega_t \varepsilon_{T,g} + \omega_s \varepsilon_{S,g})}{(\omega, \varepsilon_{R,u} + \omega_t \varepsilon_{T,u} + \omega_s \varepsilon_{S,u})} = \frac{GRFTE \left((g_g + d_g + t_g) \varepsilon_{GRFTE,g} - s_g\right)}{FTE \left((g_u + d_u + t_u) \varepsilon_{FTE,u} - s_u\right)}
\]  

(22)

Proposition 3 assumes that the university has some graduate enrolment. One special case, is where the officials of the public university places no weight placed on university size \((\omega_r = 0)\). In this case, the choice of graduate versus undergraduate scholarships depends on the weights regarding research \((\omega_r)\) and teaching \((\omega_t)\) and the effect of graduate versus undergraduate scholarships to enhance student quality and hence research and teaching \((\varepsilon_{R,g}, \varepsilon_{T,g}, \varepsilon_{R,u}, \varepsilon_{T,u})\). In addition, if the scholarships have no effect on graduate or undergraduate enrollment \((\varepsilon_{GRFTE,g} = \varepsilon_{FTE,u} = 0)\) then (22) simplifies to

22
\((\omega;_1, e_{R,sg} + \omega;_2, e_{T,sg}) + (\omega;_1, e_{R,su} + \omega;_2, e_{T,su}) = s_g GRFTE/s_u FTE\), where the right hand side is the ratio of the total graduate scholarships to total undergraduate scholarships.

### 6.0 Conclusion

While this paper provides some insight into the decision making calculus of a public or non-profit university, there are some special features of the model worth noting. First, the model assumes that universities have some expectation regarding the effect their policy choices have on its research, teaching and size. In contrast, one could view universities as being in a non-cooperative setting, in particular regarding the setting of tuition fees, admission standards, scholarships and or faculty salaries.

Specifically, each university could be making their choices simultaneously, with the overall effect on each university being influenced by the choices of other universities. In general the results of our model could be adapted to this setting, with the only significant change to the first order conditions or decision rules is that the elasticities would be “super elasticities”, that is, a university's choice of tuition, or admission standards would need to incorporate the policy responses of other universities when making its policy decisions. Alternatively we could investigate a duopoly type model, where we examine a university's policy choices given the average level of policy choices made by other universities - for example, tuition, admission standards, financial aid, and faculty numbers.

Second, given the above, it is also possible to consider a set of universities as leaders, with the other universities responding optimally to the decisions taking by these dominant universities. It is clear there are a variety of models of university systems that could be considered.

Third, an important issue that arises is to what extent do the policy choices of a public university correspond to welfare maximization. That is, is Prestige Maximization synonymous with Welfare Maximization? It is clear that the officials of public universities are in fact agents for the respective Principals who might be the elected officials, who in turn, are agents for the respective citizens. To the extent, that the “weights” chosen by University officials match the goals of the government and electorate, then this might be viewed as consistent with welfare maximization.
The closest research to the results obtained here is the work of Coates and Humphreys (2002). Both our paper and Coates and Humphreys, model the decision making of public universities, and both share some common choice variables (faculty levels). Coates and Humphreys develop both a profit maximizing and a utility maximizing model, with the latter based on a college administrator who has a utility function that depends on enrolments, the number and quality of faculty, the quality of library and laboratory facilities, the success of athletics teams and graduates, and the administrator’s income.

Our model differs from Coates and Humphreys (2002) in a number of significant ways. First, our objective function allows for a range of objectives on the part of university decision makers interested in maximizing the prestige or reputation of their university, depending on the weights placed on teaching, research and university size. Second, we show how the Niskanen case of Coates and Humphreys is a special case of our more general treatment of the preferences of university officials. Third, we are able to develop a number of useful conditions regarding the policy choices made by university officials. These can be interpreted either as normative rules, or in a positive sense, in that they describe the “implicit” weights placed on research, teaching and size, by university officials. In our case, these choices are made by university officials may or may not correspond with the respective mandates from provincial or state governments. This is similar to procedure for uncovering the social weights placed on goods by public regulators. Fourth, our model incorporates both undergraduate and graduate programs, allowing a wider range of public universities to be examined. Fifth, we assume that university officials select policy variables which effect enrolment, unlike Coates and Humphreys in which the level of enrolment is directly chosen. Our approach allows us to introduce a role for admission standards at both the undergraduate and graduate levels as an additional policy variable for university officials.

To conclude we argue in this paper that the policy tools chosen by university officials are not based solely on preferences, but must also take into account the “ability” of the university to pursue the alternative objectives of research, teaching, and size of the institution. It is clear that not all universities

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54 We do not examine the profit maximizing case of Coates and Humphreys since we are assuming that university officials for public universities cannot be viewed as a residual claimant. We do agree with Coates and Humphreys that Niskanen type behavior, in which the output of the university if expanded to the break-even point, is possible.

55 See Ross (1984 ) for a model that shows how the welfare weights that regulators place on regulated goods can be uncovered.
can or should try to mimic universities who may be in a better position to achieve one or more of the possible goals of public universities. For example, it may not be possible for all public universities to be “research universities”. We feel the approach taken here, and the decision rules obtained, may be broadly applicable to a variety of university systems.
7.0 References:


*Higher Education in Canada*, edited by Charles M. Beach, Robin W. Boadway and R. Marvin McInnis, John Deutsch Institute for the Study of Economic Policy, Queen’s University, 2005.


Appendix I. The University’s Budget

It is useful to briefly discuss the way Canadian Universities report their income and expenditures. First, universities report total university income and total university expenditure that are described as follows.

Total University Income = General Operating Income + Special Purpose & Trust + Sponsored Research + Ancillary Enterprises + Plant  
\[ (A1) \]

Total University Expenditure = General Operating Expenditure + Special Purpose & Trust + Sponsored Research + Ancillary Enterprises + Plant  
\[ (A2) \]

As can be seen the components of total university income and total university expenditure are the same. In addition, the categories often balance out, for example general operating income is close to general operating expenditure. This is similar for all other categories.

The principal difference in the categories for a university’s income and expenditure is related to the general operating categories of (A1) and (A2).

General Operating Income = Total Government Funds + Total Fees + Other Income  
\[ (A3) \]

General Operating Expenditure = Instruction and Non-Sponsored Research + Non-credit Instruction + Library + Computing + Administration and General + Physical Plant + Student Services  
\[ (A4) \]

In specifying the university’s budget constraint for the theoretical model, we simplify the budget specification somewhat. Using the General Operating categories (A3) and (A4) we abstract from Other Income and Non-Sponsored Research. The remaining expenditure categories are summarized as $3\text{EXP}_i$. 
Appendix II. List of Variables

P  Prestige (Ranking)
R  The Quality of Research Output
T  The Quality of Teaching Output
S  Size of the University (Total enrolment, ie. Full time equivalent FTET)

FTE  Full Time Equivalent course (6 credit hours)
asu  Admission Standard (Undergraduate Students)
asg  Admission Standard (Graduate Students)
GRFTE  Graduate student FTE
FTE  Undergraduate student FTE
FTET  equals  FTE + GRFTE

tu  Tuition per Undergraduate FTE
gu  Government grant per Undergraduate FTE
du  Charitable donations per Undergraduate FTE
su  Scholarships per Undergraduate FTE
tg  Tuition per Graduate FTE (GRFTE)
gg  Government grant per Graduate FTE (GRFTE)
dg  External Grants per Graduate FTE (GRFTE)
sg  Graduate funding per GRFTE
sri  Sponsored research per research faculty of quality i

L  Number of Faculty members
Lt  Number of Research Faculty members
Lt  Number of Teaching Faculty members
wr  Wage rate for research faculty members
wt  Wage rate for teaching faculty members

EXPi  Expenditure category i, includes
- Expenditure on Instruction and Non-sponsored Research
- Expenditure on Student Services
- Expenditure on the Physical Plant
- Expenditure on Administration
- Expenditure on Computing
- Expenditure on Library
- Expenditure on Non-credit Instruction per FTE