Dual Distribution and Differentiated Products

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Abstract

This paper develops an approach to analyzing the equilibrium in markets where firms selling differentiated products can choose dual distribution to sell their products. Dual distribution involves a firm selling its product both through company owned stores and through independently operated franchises. In choosing the proportion of company owned versus franchise stores, in equilibrium, the firms have no incentive to alter this ratio given the proportions chosen by rival firms. The approach taken here in analyzing dual distribution is quite general and can be applied in a variety of settings.

1

1 Introduction

It has been observed that firms in an industry often adopt alternative organizational forms. For example, in many industries, particularly the retail industry, company owned stores operate along side independently owned franchises, often called a dual distribution system. Dual distribution involves a firm selling its product both through company owned stores and through independently operated franchises. It is also important to note, that the extent of dual distribution, that is the variation in the company owned/franchisee owned ratio, varies both between and within industries and over time.

This paper develops an approach to analyzing the equilibrium in markets where firms selling differentiated products can choose dual distribution to sell their products. In developing this approach, this paper provides a simple method of characterizing a possible equilibrium in industries of this type, while also providing a methodology for evaluating alternative models of mixed organizational forms, that is, industries where different types of firms coexist in a given market.

The intuition is as follows. For example, suppose the firm initially has all company owned stores. A decision to switch one or more stores to a franchise operation, will only be done if it increases profits. In a differentiated products industry, this decision will also be partly determined by the choice of company owned versus franchise operated stores by rival firms. In order for an equilibrium to exist in the fraction of company owned versus franchise stores, it must be the case that each firm is indifferent between changing its fraction of company owned stores, given the equilibrium shares of company owned stores chosen by each firm. A particular strength of the model of dual

distribution examined here is that the dynamics of dual distribution can also be investigated.[1]

The model developed here departs in several ways from the existing literature on dual distribution. First, I examine the use of dual distribution by firms selling differentiated products in addition to the problem of defining the optimal distribution system for a monopolist. Given that most firms that use dual distribution are competing with other firms selling similar but differentiated products, this seems to be an important feature of dual distribution systems that has been overlooked. Second, given the emphasis on the roles that brand name and quality issues play in much of the theoretical and empirical models of dual distribution, a model of dual distribution based on product differentiation seems to be appropriate. Third, the predictions of the model used here are consistent with a number of results from the theoretical and empirical literature regarding dual distribution. Fourth, the model developed here allows for changes in the ratio of company owned stores based on changes to the economic environment facing firms. As the economic environment changes, the model yields testable implications regarding changes to this ratio.

This paper is organized as follows. Section 2 provides an introduction to the literature that examines the use of dual distribution by firms. Section 3 examines the use of dual distribution by a monopolist in the framework used in this paper while sections 4 and 5 outline a model of dual distribution based on differentiated products Cournot competition. This includes an examination of the strategic use of company ownership. Section 6 examines the dynamics of dual distribution model based on the model presented here, while Section 7 concludes the paper.

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[1]See Thompson (1994), Blair and Lafontaine (2005) and Lafontaine and Shaw (2005) for a discussion of the dynamics of dual distribution.

2 Dual Distribution: A Brief Literature Review

The issue of dual distribution is related to the rather extensive literature on franchising.[2] A seminal paper in this area is Mathewson and Winter (1985) who develop an incomplete contracting, principal-agent model to explain the nature of franchise contracts. Their model is based on the importance of a franchisor's brand name, and specify an agency model where the actions of franchisees must be monitored by the franchisor to prevent shirking, while also allowing for the possibility that the franchisor has less than perfect incentives to maintain the brand name. Their model makes a number of predictions regarding franchising. For example, the franchisor will monitor the contract with some probability and impose minimum quality standards, while franchisees may earn rents as incentive devices. In addition, the royalty fees in the contract will be nonlinear.[3] Their model does not consider the possibility that firm owners use both franchising and company ownership to sell their products.

There are a few formal explanations for dual distribution that have recently emerged.[4] The explanations are based largely on principal-agent models in which the firm uses franchising as a high powered incentive when that is desired by firm owners. Models within this approach, are Gallini and Lutz (1992) and Scott (1995) where firms operate some units to signal high quality. Gallini and Lutz (1992) develop a formal model of asymmetric information, which under appropriate conditions, shows that franchisors will use both franchising and company ownership to convey information about a new product. The reason is that both forms yield different benefits to firm

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[2]See Caves and Murphy (1976) and Mathewson and Winter (1985) for pioneering articles in this area.

[3]They also outline a mixed strategy outcome when contracts are incomplete but self-enforcing in monitoring, where the strategy sets are the probability that the franchisee free rides, and the probability that the franchisor inspects the site.

[4]A nice overview is found in Lafontaine and Shaw (2005).

owners. The benefit from franchising is that franchisees work harder than company managers since they have property rights over the stream of future profits. The benefit to company ownership is that franchisors can use company ownership to convince potential franchises about the profitability of franchise opportunities.

Apart from this theoretical work, there are a number of empirical papers related to the issue of dual distribution, or what is sometimes called partial vertical integration. Minkler (1990) using data based on fast food franchise outlets in Northern California and Nevada, finds that outlets farther from the firm's headquarters tend to be franchised while older firms, having more experience in a market, tend to have more company owned outlets. Minkler and Park (1994) observe that the transaction cost approach to vertical integration (Klein (1980), Rubin (1978) and Mathewson and Winter (1985a)) predicts that as the value of the firm's brand name increases, the proportion of franchised to company-owned outlets should fall. The reason is that the increased brand name provides a larger incentive for franchise owners to engage in opportunistic behavior, by cutting costs and free riding on the chain's reputation. Minkler and Park (1994) find support for the transaction cost hypotheses. In particular, they find that increases in the proportion of the brand name capital expenditures, the real interest rate, and firm growth are positively related to the increases in the degree of vertical integration, that is, company ownership. However, increases in unanticipated growth and the experience of the firm were found to be negatively related to the degree of vertical integration.

Scott (1995) also considers the mix of franchised versus company owned outlets among franchise chains. In a setting of completely homogenous retail outlets, Scott develops an empirical model to

estimate the factors that influence the proportion of franchised outlets among franchise chains. He finds that when ongoing franchisor inputs are critical to the success of a franchise system, raising the royalty rate increases the franchisor's incentives to perform, while decreasing the incentives of franchisees. Scott argues that by owning and operating the outlets themselves, the franchisor can accomplish the same objective as increasing the royalty rate.

An important empirical issue is whether there is a predictable long run relationship between the fraction of company owned versus franchise operated outlets adopted by firms. Thompson (1994) argues that franchising, or independent ownership of stores, is important for facilitating rapid growth in retail markets. He provides evidence in favour of the Penrose effect, which is the argument that there are managerial limits to growth - specifically, cost increases from too rapid an increase in staff - which can be circumvented by choosing to expand by the use of franchising.[5] Moreover, Thompson argues that if the Penrose effect declines over time, this may lead to some cyclical reversion to company ownership.

Blair and Lafontaine (2005) examine the issue of mixed organizational form or what they call dual distribution. Their data indicates that the extent of company owned versus franchisor owned stores varies by industry. They show that the variation in the company owned/franchisor owned ratio varies both between industries and within industries. Industries with a high fraction of company ownership include restaurants, retailing (non-food), convenience stores. Industries with a low fraction of company owned outlets include automobile and truck dealers, educational products and

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[5]See Edith Penrose (1959) for a presentation of this argument. Thompson (1994:211) provides a number of reasons why franchising may allow a firm to externalize part of the management function, circumventing the Penrose constraint.

services and construction, home improvement and cleaning services.[6] As pointed out by Lafontaine and Shaw (2005), most studies that use a cross-sectional approach find systematic evidence that the company-owned share decreases over time. In their work, Lafontaine and Shaw (2005) find, using an extensive panel data set, that experienced franchisors maintain a stable share of company owned stores over time; however, the targeted rate of company ownership varies across firms. They show that firms with valuable brand names have greater degrees of company ownership. They argue a valuable brand name provides greater incentives for firms to use company ownership in order to prevent free-riding on the firm's brand name by franchisees.

3   Dual Distribution: The Monopoly Case

As a benchmark, it is useful to examine the use of dual distribution by a monopolist in the framework developed in this paper. I assume the monopolist must decide on the distribution system to serve a market over which it has a monopoly. In addition, suppose the monopolist can sell its product through two alternative arrangements, company operated or franchise operated stores. The problem for the monopolist is to determine a distribution system which features $\overline{N}$ stores in total, normalized to 1, of which a fraction n are company stores, and a fraction $(1 - n)$ are franchised stores.[7]

I assume the demand and cost functions facing the monopolist differ depending on whether the market is served by a company owned versus a franchise operated stores. I assume that the market area of the monopolist if served entirely by franchise owned stores is fixed with the respective

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[6]See Blair and Lafontaine (2005; 84-85).

[7]Another possible modelling approach is to consider the monopolist's choice of a distribution system for a set on N local monopolies.

marginal cost of a franchised store is c_f. The inverse demand facing a franchise operated store is $p^f = f(q, \theta)$, where θ is a shift parameter which captures the growth in the demand for the monopolist's product.

In contrast I assume that the market area of the monopolist and marginal cost of a company owned store are functions of the fraction of company owned stores operated by the monopolist, that is $p^i = f(n, q, \theta)$ and $c^i(n)$ where q is output. The parameter θ is a shift factor, with $\partial p^i / \partial \theta \equiv p_\theta^i > 0$ and $\partial p^f / \partial \theta \equiv p_\theta^f > 0$, for convenience.

The assumption that a firm's demand is related to the fraction of company owned stores requires some justification. Here there are a number of alternative explanations. The first is to consider the fraction of company owned stores as an x variable, which shifts out the demand for the monopolist's product, specifically, $p = f(x, q)$ with $\partial p / \partial x > 0$. This modelling approach has been commonly used in models of resale price maintenance, where x is alternatively, the number of outlets, quality of the product, or special services, to name a few.[8] The second explanation can be attributed to Gallini and Lutz (1992:475) who argue their theory is consistent with a setting where firms use company owned stores to learn more about the demand for their products. Presumably this increased knowledge has the potential to shift out the firm's demand curve.[9]

The assumption that the marginal cost of a company owned store is related to the fraction of company owned stores can be justified as well. The effect the number (or fraction) of company owned firms has on demand and costs, may be thought of as a simple method of including a "Penrose" effect, which suggests there are limits to the benefits that firms realize from company

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[8]See Mathewson and Winter (1985b, 1998) for nice overviews of the resale price maintenance literature.

[9]Gallini and Lutz also point out that in this version of their theory, the signal employed would combine both company ownership of additional outlets and the use of a royalty.

ownership. This limit is related to the overall effect, benefits minus costs, of increasing the fraction of company owned stores. Specifically, the benefit from increasing the fraction of company ownership on the market size of the monopolist is assumed to be $\partial p^i/\partial n \equiv p_n^i > 0$ and $\partial^2 p^i/\partial n^2 \equiv p_{n,n}^i \leq 0$ while the effect on the cost function of a company owned store is $c_n^i > 0$ and $c_{n,n}^i \geq 0$. Specifically, the operating costs of company owned stores as a function of the fraction of company owned stores operated by a firm, are increasing at a nondecreasing rate.[10] I also assume that the fixed costs of operating a company owned versus a franchise owned firm are identical.[11] Finally, I assume that the monopolist charges the franchisee a royalty rate, r, based on sales. I assume there is a competitive market for franchises so that the monopolist must charge a competitive royalty rate.[12]

The monopolist's overall profits are then

$$\pi^M = n \cdot (p^i(n) - c^i(n))q + (1 - n) \cdot (rp^f - c^f)q \tag{1}$$

If the monopolized market is served entirely by company owned stores ($n = 1$) then $\pi^M = (p^i(n, q, \theta) - c^i(n))q$, if it is served entirely by franchise operated stores ($n = 0$) then $\pi^M = (rp^f(q, \theta) - c^f)q$. As can be seen as the monopolist increases its use of company owned stores, ($n > 0$) this shifts out the overall demand for the monopolist's product, but also brings forth additional costs, since the marginal and average costs of company owned stores is increasing in n.

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[10]What is key to the model is some asymmetry between the market areas and costs when served by company owned versus franchise operated stores. It is possible that the market area and marginal cost of franchised stores, rather than company owned stores, could be related to the fraction of company owned outlets chosen by the monopolist. Alternatively, the market area facing both company owned and franchised operated stores (as well as their marginal costs) could be functions of the fraction of company owned stores (n). Apart from the a slight change in the interpretation of the model, the principal effect is an increase in algebraic clutter in the model.

[11]The assumptions regarding fixed costs and franchise fees also apply to the oligopoly case outlined in the next section.

[12]The alternatives are for the monopolist is to charge a franchise fee, or a royalty and franchise fee. In this paper I focus on the role of the royalty rate.

I assume that the monopolist chooses its output $q*$ and distribution system $n*$ sequentially.[13]

In choosing this output, I assume that the monopolist chooses the overall output for the system.[14]

The optimal output for the monopolist is as follows

$$\partial \pi^M / \partial q = n(p^i + qp_q^i - c^i) + (1-n)[(r(p^f + qp_q^f) - c^f)] \leq 0, if < 0, q = 0 \qquad (2)$$

where $p_q^i \equiv \partial p^i / \partial q$ and $p_q^f \equiv \partial p^f / \partial q$. The second order condition governing the optimal choice of

q is $D \equiv (n(2p_q^i + qp_{q,q}^i) + (1-n)[(r(2p_q^f + qp_{q,q}^f)] < 0$ for a maximum.

Proposition 1 For a monopolist choosing a dual distribution system, the optimal output of the

monopolist, $q*$, (i) increases with the royalty rate paid by franchised stores (r), (ii) decreases with

the marginal costs of franchised stores (c_f) and(iii) increases with the market size of the monopolist

(θ).

Proof: Taking the total differential of (2) yields $[n(p_n^i + qp_{q,n}^i - c_n^i) + (p^i + qp_q^i - c^i) - (r(p^f +$

$qp_q^f) - c^f)]dn + [n(2p_q^i + qp_{q,q}^i) + (1-n)(r(2p_q^f + qp_{q,q}^f)]dq + [(np_\theta^i + qp_{q,\theta}^i) + (1-n)(r(p_\theta^f + qp_{q,\theta}^f]d\theta +$

$(1-n)(p^f + qp_q^f)dr - (1-n)dc^f = 0$. To determine (i)$dq*/dr = -(1-n)(p^f + qp_q^f)/D > 0$. (ii)

$dq*/dc^f = (1-n)/D < 0$ and (iii) $dq*/d\theta = -[(np_\theta^i + qp_{q,\theta}^i) + (1-n)(r(p_\theta^f + qp_{q,\theta}^f]/D > 0$ where

$D \equiv (n(2p_q^i + qp_{q,q}^i) + (1-n)[(r(2p_q^f + qp_{q,q}^f)] < 0.\|$

Given the optimal output of the monopolist, the optimal fraction of company owned stores for

the monopolist $(n*)$ can be determined by taking the derivative of (1), recognizing that the optimal

[13]The case of a simultaneous choice of output and and distribution network has been examined, and is available on request.

[14]Thus,I am ruling out the possibility that the monopolist might use quantity or price discrimination with franchise operated stores. That is, I am assuming that the monopolist cannot set different prices for franchise owned versus company owned stores. I feel this is a reasonable assumption, although a quantity discrimination model might be an interesting extension to the model developed here.

output $q*$ is determined by $q* = f(n, r, c^f, \theta)$, from condition (2). Taking the derivative and using the envelope theorem and setting equal to zero, yields

$$\partial \pi^M / \partial n = [n(p_n^i - c_n^i) + (p^i - c^i)]q - (rp^f - c^f)q \leq 0, if < 0, n = 0 \tag{3}$$

There is an interesting interpretation of (3). One can think of the term $MR(n) = [n(p_n^i - c_n^i) + (p^i - c^i)]q$ as the marginal profit from increasing the share of company owned stores (n), while the term $MC(n) = (rp^f - c^f)q$ is the marginal cost, that is the loss in profits from reducing the use of franchised stores. An interior solution requires the $MR(n)$ from company ownership equal the $MC(n)$.

In order for the share of company owned stores given by (3) to be a maximum, we need the second order condition, $D \equiv n(p_{n,n}^i - c_{n,n}^i) + 2(p_n^i - c_n^i) < 0$. The second order condition requires some intuition. Ignoring the first term in D, $(p_{n,n}^i - c_{n,n}^i)$, the second order condition requires $(p_n^i - c_n^i) < 0$. For stability, this means that when the number of company owned stores used by a monopolist increases, the difference between the market size and marginal cost of company owned stores $(p^i(n) - c^i(n))$ must decrease when n increases.

We can also investigate how changes in the royalty rate, costs and market size facing the monopolist effect the number of company owned stores in the dual distribution system, $n*$.

Proposition 2 For a monopolist, choosing a dual distribution system, the optimal fraction of company owned stores $(n*)$ (i) decreases with the royalty rate paid by franchised stores (r), (ii) increases with the marginal costs of franchised stores (c^f), (iii) increases with the market size of the monopolist (θ).

11

Proof: (i)Straightforward comparative statics of (3) yields regarding the fraction of company owned stores $n*$ chosen by a monopolist (i) $dn * /dr = p^f /D < 0$ and (ii) $dn * /dc^f = -1/D > 0$ and (iii) $dn * /d\theta = -[np^i_{n,\theta} + p^i_\theta + p^f_\theta]/D > 0$, given $D \equiv n(p^i_{n,n} - c^i_{n,n}) + 2(p^i_n - c^i_n) < 0$. $\|$.

In terms of Proposition 2, the comparative static (i) is consistent with the literature, in that a higher royalty rate paid by franchised stores reduces the benefit of company owned stores. Regarding (ii), higher marginal costs of franchised outlets, also increase the benefit and use of company owned stores. Of interest is (iii) which indicates as the demand for the monopolist's product grows, the use of company owned stores increases (unless $np^i_{n,\theta} < 0$ and large).[15]

A specific functional form for the respective inverse demand functions can help to outline the nature of the equilibrium for the monopolist using a dual distribution system. For the monopolist, assume the inverse demand facing a company owned store is $p^i = \theta + \alpha^i(n) - \beta q$ while for a franchised outlet it is $p^f = \theta + \alpha^f - \beta q$. Using (2) we can solve for the optimal output and profit for the monopolist,

$$q* = \frac{n[\theta + \alpha(n) - c(n)] + (1 - n)[r(\theta + \alpha_f) - c_f]}{2\beta}$$

$$\pi* = \frac{[n(\theta + \alpha(n) - c(n)) + (1 - n)(r(\theta + \alpha_f) - c_f)]^2}{4\beta}$$

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[15] If the sign is negative, this means that an increased demand for the monopolist's product reduces the demand increasing effect of company owned stores.

4 Dual Distribution and Differentiated Products

As has been discussed, the use of franchised outlets is common in industries in which firms sell related brand name products. In order to address this issue, I consider the role that differentiated products competition has on the existence of a dual distribution equilibrium. Specifically, the demand functions facing the two Firms are given as $p^{1i} = f(q_1, q_2, n_1, \theta_1)$ and $p^{1f} = f(q_1, q_2, \theta_1)$ for Firm 1 and $p^{2i} = f(q_1, q_2, n_2, \theta_2)$ and $p^{2f} = f(q_1, q_2, \theta_2)$ for Firm 2. For the case of substitutes, the required derivatives are $\partial p^{1i}/\partial q_2 < 0$ and $\partial p^{1f}/\partial q_2 < 0$ and $\partial p^{2i}/\partial q_1 < 0$ and $\partial p^{2f}/\partial q_1 < 0$.[16]

I assume that company owned and franchised outlets have different costs. I denote the variable costs of the company stores for Firm 1 is $c^{1i}(n_1)$ while the variable costs for company owned stores for Firm 2 are $c^{2i}(n_2)$. In contrast, the variable costs of providing the product through a franchised store (or franchised outlet) for both firms is c^f.[17]

For Firm 1, consider the decision to have a dual distribution system, that is to sell some fraction of its output through company owned stores, $n_1 > 0$. The profits from this strategy are

$$\pi_1 = n_1 \cdot (p_{1i}(n_1) - c^i(n_1))q_1 + (1 - n_1) \cdot (r_1 p_{1f} - c^f)q_1 \tag{4}$$

where $p_{1i}(p_{1f})$ are the respective prices of firm 1's product sold by an company owned versus a franchised store, and r_1 is the royalty rate paid by the franchised store. The corresponding profit function for Firm 2 is

$$\pi_2 = n_2 \cdot (p_{2i}(n_2) - c^i(n_2))q_2 + (1 - n_2) \cdot (r_2 p_{2f} - c^f)q_2 \tag{5}$$

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[16]It is important to note that we are ruling out price or quantity discrimination between company owned and franchise operated stores.

[17]I abstract from the fixed costs of operating the company owned or franchised outlets.

As for the case of a monopolist firm, the assumption is that each firm must decide on what fraction of stores to operate as company owned stores. We are now in a position to determine the conditions under which a dual distribution exists.

5 Dual Distribution Equilibrium

Here we consider the use of dual distribution in a duopoly model where firms are selling differentiated products. Similar to the monopoly setting, we assume a two stage equilibrium, where the outputs of the Firms are determined in the second stage, given the choice of distribution - the share of company owned versus franchise operated stores - made in the first stage.[18] As is customary, we solve the second stage first.

5.1 Optimal Outputs

In the second stage of the two stage model, we determine the equilibrium outputs for the two firms, given the fraction of company owned stores (n_1*, n_2*) chosen by both firms in the first stage.

Using (4) and (5) we can determine the conditions governing the optimal output chosen by Firm 1 and Firm 2, where each firm has the option of selling through a company owned or independent store.

The first order condition for Firm 1 is

$$\partial \pi^M / \partial q_1 = n_1[p^{1i} + q_1 p_{q1}^{1i} - c^{1i}] + (1 - n_1)[(r_1(p^{1f} + q_1 p_{q1}^{1f}) - c^f] \leq 0, if < 0, q_1 = 0 \qquad (6)$$

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[18]Similar to the monopoly case, we assume that two firms take the royalty rate they can charge franchise operated stores as given.

The first order condition for Firm 2 is

$$\partial \pi^M / \partial q_2 = n_2 [p^{2i} + q_2 p_{q2}^{2i} - c^{2i}] + (1 - n_2)[(r_2(p^{2f} + q_2 p_{q2}^{2f}) - c^f)] \leq 0, if < 0, q_2 = 0 \qquad (7)$$

where we have assumed Cournot conjectures, which means the respective derivatives $\partial q_1 / \partial q_2 = 0$

in (4) and $\partial q_2 / \partial q_1 = 0$ in (5) are equal to zero.

Using (6) and (7), one can solve for the respective outputs of the two firms, conditional on

each firm having some fraction of their stores company owned, $n_1 > 0$ and $n_2 > 0$, which yields

$q^1 = f(n_1, n_2, \theta_1, \theta_2, c^f, r_1, r_2)$ and $q^2 = f(n_1, n_2, \theta_1, \theta_2, c^f, r_1, r_2)$.

5.2   Optimal Distribution System

Given the optimal output in the second stage it is straightforward to determine the optimal distri-

bution system for the oligopoly case. Taking the derivative of (4) with respect to n_1 and using the

envelope theorem and setting equal to zero, yields

$$\partial \pi_1 * / \partial n_1 = [n_1(p_{n_1}^{1i} - c_{n_1}^i) + (p^{1i} - c^i)]q_1 - (r_1 p^{1f} - c^f)q_1 = 0 \qquad (8)$$

Note that (8) for the oligopoly case is similar to (3) for the monopolist, however, the optimal choice

of company owned stores $(n_1 *)$ will be determined simultaneously with the choice of company

owned stores $(n_2 *)$ by Firm 2. The corresponding condition for Firm 2 is

$$\partial \pi_2 * / \partial n_2 = [n_2(p_{n_2}^{2i} - c_{n_2}^i) + (p^{2i} - c^i)]q_2 - (r_1 p^{2f} - c^f)q_2 = 0 \qquad (9)$$

5.3   Specific Functional Forms

It is helpful to use a specific functional form for the inverse demand functions, for the following

Proposition. For Firm 1, the inverse demand facing a company owned store is $p^{1i} = \theta_1 + \alpha^i(n_1) -$

$\beta_1 q_1 - \gamma q_2$ while for a franchised outlet it is $p^{1f} = \theta_1 + \alpha^{if} - \beta_1 q_1 - \gamma q_2$. For Firm 2, the inverse demand facing a company owned store is $p^{2i} = \theta_2 + \alpha^i(n_2) - \beta_2 q_2 - \gamma q_1$ and for a franchised outlet, $p^{2f} = \theta_2 + \alpha^{if} - \beta_2 q_2 - \gamma q_1$.

Given these inverse demand functions is straightforward to derive the best response functions for these two firms. For Firm 1, it is

$$R_1(q_2) = \frac{A_1}{2\beta_1} - \frac{\gamma q_2}{2\beta_1}$$

and for Firm 2 (written in inverse form it is

$$R_2(q_2) = \frac{A_2}{\gamma} - \frac{2\beta_2 q_2}{\gamma}$$

where $A_1 \equiv n_1(\theta_1 + \alpha_1(n_1) - c^{1i}(n_1)) + (1 - n_1)(r_1(\theta_1 + \alpha_{1f})) - c_f)$ and $A_2 \equiv n_2(\theta_2 + \alpha_2(n_2) - c^{2i}(n_2)) + (1 - n_2)(r_2(\theta_2 + \alpha_{2f})) - c_f)$.

Proposition 3 Given differentiated products and Cournot competition, the optimal output for Firm 1 (q_1*) is (i) increasing with the royalty rate received from its franchised stores (r_1), (ii) decreasing with the royalty rate paid by franchised stores of the rival firm (r_2),(iii) decreasing with the growth in demand of its rival firm (θ_2), (iv) decreasing as β_2, the slope of the inverse demand curve facing its rival firm decreases and (v) increasing with the number of company owned stores operated by its rival firm (n_2).

Proof: It is straightforward to work with the best response functions for the two firms, which are given by $R_1(q_2)$ and $R_2(q_2)$. For differentiated products Cournot competition, the two goods are strategic substitutes, yielding downward sloping best response functions. Propositions (i), (ii), (iii) and (iv) can be easily illustrated using Figure 1, given A_1 and A_2 defined above. Regarding (v)

16

it is straightforward to take the total differentials of (8) and (9) for the specific demand functions, which yields the $\partial q_1*/\partial n_2 = -\gamma[n_2(\alpha_2'' - c_2'') + 2(\alpha_2' - c_2')]/(1 - r_2)(\beta_1\beta_2 - \gamma_1\gamma_2)$ Given that the denominator is positive (since $\beta_1\beta_2 > \gamma_1\gamma_2 > 0$ for stability), and the term in square brackets in the numerator is negative (by the second order condition for optimal n_2), the overall sign of $\partial q_1*/\partial n_2$ is positive.‖

[Figure 1 about here.]

Given the two best response functions it is straightforward to solve for the respective output levels for the specific functional form case

$$q_1* = \frac{2A_1\beta_2 - \gamma A_2}{D}$$

and

$$q_2* = \frac{2A_2\beta_1 - \gamma A_1}{D}$$

where $D \equiv 4\beta_1\beta_2 - \gamma^2$.

Or for the symmetric case, $A1 = A_2 = A = n(\theta + \alpha(n) - c^i(n)) + (1 - n)(r(\theta + \alpha_f)) - c_f)$

$$q_c* = \frac{n(\theta + \alpha(n) - c^i(n)) + (1 - n)(r(\theta + \alpha_f)) - c_f)}{(2\beta + \gamma)}$$

5.4 Strategic Use of Dual Distribution

Now given the optimal outputs chosen by the respective firms in the second stage (which are dependent on the fraction of company owned stores n_1 and n_2 chosen in the first stage), we can then determine the optimal fraction of company owned stores that would be chosen by Firm 1 in the strategic case.

Using the profit function given by (4), and the first order condition (6) and the envelope theorem, the optimal choice of company owned stores for Firm 1 is determined as follows.

$$\partial \pi_1 * / \partial n_1 = [n_1(p_{n_1}^{1i} - c_{n_1}^i) + (p^{1i} - c^i)]q_1 - (r_1 p^{1f} - c^f)q_1 + q_1[n_1 p_{q_2}^{1i} + (1 - n_1)p_{q_2}^{1f}]\partial q_2 / \partial n_1 \quad (10)$$

The first two terms in (10) are similar to condition (3) for the monopoly case. However, in the differentiated products oligopoly case there is an additional term $q_1[n_1 p_{q_2}^{1i} + (1 - n_1)p_{q_2}^{1f}]\partial q_2 / \partial n_1$. This can be termed the strategic effect and captures the effect of an increase in the fraction of company owned stores chosen by Firm 1 on Firm 2's output (q_2), and subsequently Firm 1's prices (p^{1i} and p^{1f}) and hence the profits of Firm 1.

Proposition 4. Given differentiated products and Cournot competition, (i) the fraction of company owned stores chosen by firms in the strategic case differs from the non-strategic case. That is $n_1 * \neq n_{1s}*$, i=1,2, where $n_1 *$ is the fraction of company owned stores chosen in the non-strategic case, and $n_{1s}*$ is the fraction of company owned stores chosen by a strategic Firm, (ii) the share of company owned stores, n_i is lower in the strategic case than in the non-strategic case n_{is}.

Proof: (i) In the non-strategic case $[n_1 p_{q_2}^{1i} + (1 - n_1)p_{q_2}^{1f}]\partial q_2 / \partial n_1 = 0$ which results in a different choice of company owned stores than in the strategic case. In order to prove (ii), we make use of

18

Proposition 3 (v), which indicates that $\partial q_1/\partial n_2 > 0$, and given symmetry implies that $\partial q_2/\partial n_1 > 0$. Given that the two goods are substitutes, $p_{q_2}^{1i} < 0$ and $p_{q_2}^{1f} < 0$, the entire term $[n_1 p_{q_2}^{1i} + (1 - n_1)p_{q_2}^{1f}]\partial q_2/\partial n_1 < 0$, which means that the marginal benefit of company owned stores is reduced, lowering the fraction of company owned stores chosen by Firm 1, n_1*.$\|$

To summarize, the equilibrium fraction of company owned stores in the non-strategic case can be determined by solving (8) and (9). For the strategic case, with Firm 1 being the leader (10) and (9) are solved simultaneously.

5.5 Empirical Implications

The model developed here suggests the equilibrium fractions of company owned stores (n_1*, n_2*) are functions of the respective costs c_f and market size shift parameters θ_1 and θ_2 respective royalty rates (r_1, r_2) and the product differentiation parameter γ.

Given the above model, it is clear that the for the differentiated products model, the empirical model is of the form,

$$n_1 = f[r_1, r_2, \theta_1, \theta_2, c_f, \gamma] + \varepsilon \tag{11}$$

$$n_2 = g[r_1, r_2, \theta_1, \theta_2, c_f, \gamma] + \varepsilon \tag{12}$$

While it is possible to consider alternative explanations for dual distribution, or to use different models of product differentiation, the approach taken here leads naturally to an empirical formulation.

6 The Dynamics of Dual Distribution

Given the analysis developed in this paper, it is clear that the choice between organizational forms, within a differentiated products model, has the form of a noncooperative game between firms. In this section, we develop this idea further in the context of a normal form depiction of the dual distribution decision. In the following table, we have two strategies for the respective firms, Firm 1 and Firm 2. Each firm can choose to sell their products using company owned stores or franchisor owned stores. The payoffs in the table are profits from the various combinations of the respective profits that result from the competition between company owned and franchised stores for the two firms. Company owned stores are identified as (c), whereas franchised stores are identified as (f).[19] The first (second) entry in the superscript for each cell, identifies the profit level of Firm 1 (Firm 2). The payoffs for the two firms are written as $\pi_i(s_i, s_j)$ and $\pi_j(s_i, s_j)$ when Firm i chooses strategy s_i where $s_i = c, f$ and Firm j chooses s_j where $s_j = c, f$, where $c \equiv$ Company Owned and $f \equiv$ Franchised stores.

<center>Firm 2</center>

		Company	Franchised
Firm 1	Company	$\pi_1^{c,c}, \pi_2^{c,c}$	$\pi_1^{c,f}, \pi_2^{c,f}$
	Franchised	$\pi_1^{f,c}, \pi_2^{f,c}$	$\pi_1^{f,f}, \pi_2^{f,f}$

It is clear that if $\pi_1^{c,c} > \pi_1^{f,c}$ and $\pi_2^{c,c} > \pi_2^{c,f}$ then there is a Nash equilibrium with both firms

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[19]There is a literature which examines competition between vertically integrated firms and vertically separated firms (which feature independent manufacturers and retailers). See Bonnano and Vickers (1988) and Cyrenne (1994) for models addressing this issue.

using only Company Owned stores. Similarly, if $\pi_1^{f,f} > \pi_1^{c,f}$ and $\pi_2^{f,f} > \pi_2^{f,c}$ then there is a Nash equilibrium with both firms only using Franchise operated stores.

It is also possible that there is what I call a dual distribution equilibrium, (n_1*, n_2*), in which both firms use both company owned and franchise operated stores.[20] To determine the conditions under which a dual distribution equilibrium exists we proceed as follows. If we assume that Firm 1 chooses the share of Company Owned (Franchised Owned) as $n_1(1 - n_1)$ and Firm 2 chooses the Company Owned (Franchised Owned) with shares $n_2(1 - n_2)$, then the the condition of a dual distribution equilibrium requires that given the shares of company owned versus franchise stores operated by the rival firms, that the change in profits from altering the share of company owned versus franchise operated stores must be zero. This equilibrium yields the respective shares for company owned stores for each firm as (n_1*, n_2*). Specifically, it must be true for the shares (n_1*, n_2*) to be an equilibrium, the following condition must hold for Firm 1 (with a similar condition for Firm 2) $n_2(\pi_1^{c,c}) + (1 - n_2)(\pi_1^{c,f}) = n_2(\pi_1^{f,c}) + (1 - n_2)(\pi_1^{f,f}) = 0$, or

$$n_2(\pi_1^{c,c} - \pi_1^{f,c}) + (1 - n_2)(\pi_1^{c,f} - \pi_1^{f,f}) = 0 \tag{13}$$

where the first term is expected profits to Firm 1 from increasing the use of company owned stores (given Firm 2 is using a given fraction of company owned stores) while the second term is the expected profits from increasing the used of company owned stores (given Firm 2 is using a given fraction of franchise operated stores.)

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[20]It is clear, given that $0 \leq n_1 \leq 1$ and $0 \leq n_2 \leq 1$, the dual distribution case is similar to what might be considered a "mixed strategy" equilibrium to a game played between firms in choosing its dual distribution system. One significant difference is that in a mixed strategy equilibrium as usually understood, results in either a company owned or a franchise operated store being chosen, rather than a "fraction" of company owned stores.

We can rearrange equation (10) to express n_2 the share of owned stores chosen by Firm 2 as

$$n_2 = [\pi_1^{f,f} - \pi_1^{c,f}]/[(\pi_1^{f,f}) - \pi_1^{c,f}) + (\pi_1^{c,c} - \pi_1^{f,c})] \qquad (14)$$

The numerator of (11) is the expected profit gain to Firm 1 from increasing the use of franchised operated stores versus company owned stores (given that Firm 2 has chosen to use a given fraction of franchised owned stores). The denominator involves 2 terms in brackets, the first is, once again, the expected profit gain to Firm 1 from increasing its use of franchised stores versus company owned stores (given that Firm 2 has chosen to use a given fraction of franchised owned store). The second term is the profit gain to Firm 1 from increasing the use of company owned stores (given that Firm 2 has chosen to operate a given fraction of outlets as company owned stores.) The corresponding condition for n_1, the share of company owned stores chosen by Firm 1 is

$$n_1 = [\pi_2^{f,f} - \pi_2^{c,f}]/[(\pi_2^{f,f}) - \pi_2^{c,f}) + (\pi_2^{c,c} - \pi_2^{f,c})] \qquad (15)$$

It is possible to rewrite the payoff Table as two separate matrices, where is A (B) is the payoff matrix associated with Firm 1 (Firm 2).

$$A = \begin{pmatrix} \pi_1^{c,c} & \pi_1^{c,f} \\ \pi_1^{f,c} & \pi_1^{f,f} \end{pmatrix} \qquad B = \begin{pmatrix} \pi_2^{c,c} & \pi_2^{c,f} \\ \pi_2^{f,c} & \pi_2^{f,f} \end{pmatrix}$$

As outlined by Weibull (1995), we can classify games games depending on the relationship of the entries in the two matrices for the two players. If $B^T = A$, we have a symmetric game, while if $B = A$, we have a doubly symmetric game.[21] Initially, we will consider a symmetric game, in that

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[21] Weibull (1995:25,26). For an alternative treatment of a number of similar issues, see Cressman (2003).

$\pi_1^{f,c} = \pi_2^{c,f}$ and $\pi_1^{c,f} = \pi_2^{f,c}$ this means that $B^T = A$. As outlined by Weibull (1995), we can also determine the nature of the Nash equilibrium by rewriting the above matrix to yield the matrix C which is equivalent to matrix A.

$$C = \begin{pmatrix} \pi_1^c & 0 \\ 0 & \pi_1^f \end{pmatrix}$$

where $\pi_1^c = \pi_1^{c,c} - \pi_1^{c,f}$ and $\pi_1^f = \pi_1^{f,f} - \pi_1^{f,c}$.

Given the payoff equivalent matrix C, we can rewrite (12) in terms of the payoff equivalent cells as

$$n_{2*} = \pi_1^f / [\pi_1^f + \pi_1^c] \tag{16}$$

We can repeat the same exercise for Firm 2, which will yield the respective shares, $(n_1, (1-n_1))$, that Firm 1 chooses a company owned versus and franchised store. The dual distribution equilibrium in a differentiated markets involves the pair (n_1*, n_2*) which are the respective shares of company owned stores chosen by Firm 1 and Firm 2.

Given the symmetric nature of the problem, we can focus on Firm one's problem. We can define strategy c (f) as the choice of company owned (independent operated) stores by firms. We can also define n_1 $(1 - n_1)$ as the share of firms choosing company owned stores (franchised stores). Substituting for the elements in the C matrix which is equivalent to the A matrix defined above,

$$C = \begin{pmatrix} \pi_1^c & 0 \\ 0 & \pi_1^f \end{pmatrix}$$

which means that $a_1 = \pi_1^c$ and $a_2 = \pi_1^f$ yields

$$\dot{n}_1 = [\pi_1^c n_1 - \pi_1^f (1 - n_1)] n_1 (1 - n_1) \tag{17}$$

23

The dynamics of the system are then determined by the signs of the elements of the C matrix outlined above, where $\pi_1^c = \pi_1^{c,c} - \pi_1^{c,f}$ and $\pi_1^f = \pi_1^{f,f} - \pi_1^{c,f}$.[22] There are two general cases to consider which are dependent on the sign of the expression in the square bracket of (21).

(i) Case 1

If the product is less than zero, ie. $(\pi_1^c \pi_1^f < 0)$ then the population share n_1 always declines (when $\pi_1^c < 0$ and $\pi_1^f > 0$) or always grows (when $\pi_1^c > 0$ and $\pi_1^f < 0$). The resulting is a corner solution for the population share x_1.

(ii) Case 2

If the product is greater than zero, ie. $(\pi_1^c \pi_1^f > 0)$, then the growth rate of n_1 changes sign when $a_1 n_1 = a_2(1 - n_1)$, which occurs precisely at the dual distribution equilibrium value $n_1 = \lambda = a_2/(a_1 + a_2)$ or $n_1 = \pi_1^f/(\pi_1^c + \pi_1^f)$. Two sub-cases are possible here. If both payoffs are positive, then n_1 decreases toward 0 from any initial value $n_1^o < \lambda$ and toward 1 from any initial value $x_1^o > \lambda$. If both payoffs are negative, then the population share n_1 increases toward λ from any lower initial value and decreases toward λ from any higher initial value. It is only in this latter case, that the population share n_1 is stable $0 < n_1 < 1$.

In order to determine whether a dual distribution equilibrium exists, one must determine the respective magnitudes of the payoff equivalent profits given by (18). It is clear however, that the respective payoffs depend on the nature of the competition that exists between a company owned store and and independently owned and operated outlet. While there are a variety of different types

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[22]The following dynamic analysis is based on the idea of replicator dynamics. A nice treatment is Weibull (1995:77).

of competition that could be considered, in this paper, I consider the role of product differentiation, specifically, the case of dual distribution where firms are selling differentiated products.

6.1   Dynamics of Dual Distribution: Other Issues

Given the above, the question is whether a stable dual distribution system exists, for the differentiated products Cournot model examined earlier. To answer this question, we must evaluate $\pi_1^c = \pi_1^{c,c} - \pi_1^{c,f}$ and $\pi_1^f = \pi_1^{f,f} - \pi_1^{f,c}$. If it can be shown that in general $\pi_1^c < 0$ and $\pi_1^f < 0$, then a stable equilibrium exists which features a stable share of company owned stores operated by firms.

Given the above results, it is clear that a stable long run dual distribution system may or may not exist depending on the profit levels π_1^c and π_1^f. These profit levels may differ depending on the type of differentiated product model that is used to analyze the competition between firms. In addition, the stability of the dual distribution system, can also be affected by important asymmetries in the differentiated products model, that is, differences in α_{ks}, α_{ki} and c_{ks}, c_{ki} and $k = 1, 2$. These asymmetries can affect the values of π_1^c and π_1^f and the stability of dual distributions systems.[23] Moreover, different explanations for dual distribution, other than differentiated products competition, may give rise to different profit levels in the C matrix, which would give yield differences in the stability of the respective dual distribution systems.

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[23]Asymmetric models will require in general simulations to determine the relative importance of these asymmetries for the long run dynamics.

7   Conclusions

This paper has presented a relatively simple framework for addressing the question of dual distribution by firms which occurs in a number of industries.

A number of features of dual distribution are included in the model developed here. Consistent with the emphasis in the literature on role of brand names, we incorporate differentiated products competition between firms. We also include a motivation for limits to the fraction of company owned stores used by firms as suggested by the "Penrose Effect". Finally, we link the static model to a method that can analyze the dynamics of dual distribution which has been an important issue in the literature.

A principal contribution of the paper is to develop a methodology to analyze the use of dual distribution by firms. The explanation I emphasize is based on role of differentiated products competition between firms. A particular strength of the framework developed here is that in incorporates a number of key elements of dual distribution, while also providing a method of investigating the stability of dual distribution systems used by firms.

8 References

[1] Blair, Roger D. and Francine Lafontaine *The Economics of Franchising*, New York: Cambridge, 2005.

[2] Bonanno, Giacomo and John Vickers (1988) "Vertical Separation" *Journal of Industrial Economics*, 36, 257-265.

[3] Caves, R.E. and W.F. Murphy (1976) "Franchising: Firms, Markets and Intangible Assets," *Southern Economic Journal*, Vol. 42, 572-586.

[4] Cressman, Ross *Evolutionary Dynamics and Extensive Form Games*, Cambridge: The MIT Press, 2003.

[5] Cyrenne, Philippe (1994) "Vertical Integration versus Vertical Separation" *Review of Industrial Organization*, 9, 311-322.

[6] Gallini, Nancy and Nancy A. Lutz (1992) "Dual Distribution and Royalty Fees in Franchising" *The Journal of Law, Economics and Organization*, 8, 471-501.

[7] Hofbauer, Josef and Karl Sigmund (1998) *Evolutionary Games and Population Dynamics*, Cambridge: Cambridge University Press.

[8] Lafontaine, Francine and Kathryn L. Shaw (2005) "Targeting Managerial Control: Evidence from Franchising" *The RAND Journal of Economics* Vol. 36, No. 1, 131-150.

[9] Mathewson, F.G. and R.A. Winter (1985a) "The Economics of Franchise Contracts," *Journal of Law and Economics*, Vol. 28, pp. 503-526.

[10] Mathewson, G. Frank and Ralph A. Winter (1985b) Competition Policy and Vertical Exchange Ottawa: Minister of Supply and Services.

[11] Mathewson, Frank and Ralph Winter (1998) "Law and Economics of Resale Price Maintenance" Review of Industrial Organization, 13, 57-84.

[12] Minkler, Alanson P. and Timothy A. Park (1994) "Asset Specificity and Vertical Integration in Franchising" *Review of Industrial Organization*, 9, 409-423.

[13] Minkler, Alanson P. (1990) "An Empirical Analysis of a Firm's Decision to Franchise" *Economics Letters*, 34, 77-82.

[14] Motta, Massimo *Competition Policy: Theory and Practice*, Cambridge: Cambridge University Press, 2004

[15] Penrose, Edith T. *The Theory of the Growth of the Firm*, Oxford: Basil Blackwell, 1959.

[16] Scott, Frank A. Jr. (1995) "Franchising vs. Company Ownership as a Decision Variable of the Firm" *Review of Industrial Organization*, 10: 69-81.

[17] Shy, Oz *Industrial Organization: Theory and Applications*, Cambridge: The MIT Press, 1995.

[18] Thompson, R. Steve (1994) "The franschised life cycle and the Penrose effect" *Journal of Economic Behavior and Organization*, 24: 207-218.
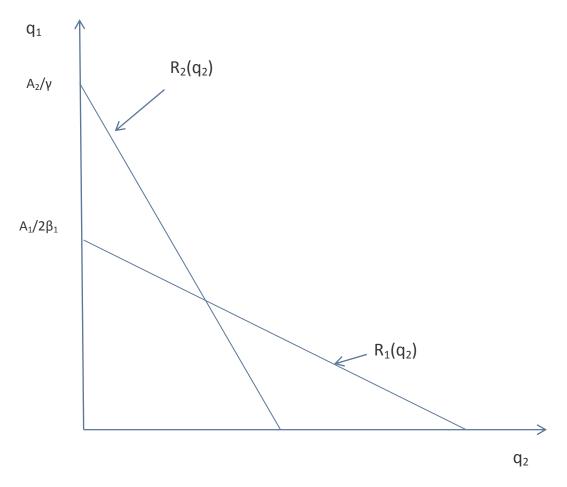
[19] Weibull, Jörgen W. *Evolutionary Game Theory*, Cambridge: The MIT Press, 1995.

Figure 1: Best Response Functions